Newton's Rapheson Method (N-R Method)

Algorithm

Newton Raphson method Steps (Rule)	
Step-1:	Find points a and b such that $a<b$ and $f(a)\cdot f(b)<0$.
Step-2:	Take the interval $[a,b]$ and find next value $x0=a+b2$
Step-3:	Find $f\left(x0\right)$ and $f'\left(x0\right)$ $x1=x0-f\left(x0\right)f'\left(x0\right)$
Step-4:	If $f\left(x1\right)=0$ then $x1$ is an exact root, else $x0=x1$
Step-5:	Repeat steps 2 to 4 until $f\left(xi\right)=0$ or $\left\|f\left(xi\right)\right\| \leq \text{Accuracy}$

Example-1

Find a root of an equation $f(x)=x^3-x-1$ using Newton Rapheson method

Solution:

Here $x^3-x-1=0$

Let $f(x)=x^3-x-1$

$\therefore f'(x)=3x^2-1$

Here

x	0	1	2
$f(x)$	-1	-1	5

Here $f(1)=-1<0$ and $f(2)=5>0$

$\therefore$ Root lies between 1 and 2

$x0=1+22=1.5$

$1st$ iteration :

$f\left(x0\right)=f(1.5)=0.875$

$f'\left(x0\right)=f'(1.5)=5.75$

$x1=x0-f\left(x0\right)f'\left(x0\right)$

$x1=1.5-0.8755.75$

$x1=1.34783$

2_{nd} iteration :

$f\left(x_1\right)=f(1.34783)=0.10068$

$f'\left(x_1\right)=f'(1.34783)=4.44991$

$x_2=x_1-f\left(x_1\right)f'\left(x_1\right)$

$x_2=1.34783-0.100684.44991$

$x_2=1.3252$

3_{rd} iteration :

$f\left(x_2\right)=f(1.3252)=0.00206$

$f'\left(x_2\right)=f'(1.3252)=4.26847$

$x_3=x_2-f\left(x_2\right)f'\left(x_2\right)$

$x_3=1.3252-0.002064.26847$

$x_3=1.32472$

4_{th} iteration :

$f\left(x_3\right)=f(1.32472)=0$

$f'\left(x_3\right)=f'(1.32472)=4.26463$

$x_4=x_3-f\left(x_3\right)f'\left(x_3\right)$

$x_4=1.32472-04.26463$

$x_4=1.32472$

Approximate root of the equation $x_3-x-1=0$ using Newton Raphson mehtod is 1.32472

n	x_0	$f\left(x_0\right)$	$f'\left(x_0\right)$	x_1	Update
1	1.5	0.875	5.75	1.34783	$x_0=x_1$
2	1.34783	0.10068	4.44991	1.3252	$x_0=x_1$
3	1.3252	0.00206	4.26847	1.32472	$x_0=x_1$
4	1.32472	0	4.26463	1.32472	$x_0=x_1$