Iteration Method

Algorithm

Iteration method Steps (Rule)	
Step-1:	First write the equation $x=\phi(x)$
Step-2:	Find points a and b such that $a<b$ and $f(a) \cdot f(b)<0$.
Step-3:	If $f(a)$ is more closer to 0 then $f(b)$ then $x0=a$ else $x0=b$
Step-4:	$x1=\phi\left(x0\right)$ $x2=\phi\left(x1\right)$ $x3=\phi\left(x2\right)$ ... Repeat until $\left\|f\left(xi\right)-f\left(xi\text{-}1\right)\right\|\approx 0$

Example-1
Find a root of an equation $f(x)=x^3-x-1$ using Iteration method

Solution:
Method-1
Let $f(x)=x^3-x-1$

$x^3-x-1=0$

$\therefore x^3=x+1$

$\therefore x=\sqrt[3]{x+1}$

$\therefore \phi(x)=\sqrt[3]{x+1}$

Here

x	0	1	2
$f(x)$	-1	-1	5

Here $f(1)=-1<0$ and $f(2)=5>0$

$\therefore$ Root lies between 1 and 2

$x0=1+22=1.5$

$x1=\phi\left(x0\right)=\phi(1.5)=1.35721$

$x2=\phi\left(x1\right)=\phi(1.35721)=1.33086$

$x3=\phi\left(x2\right)=\phi(1.33086)=1.32588$

$x_4=\phi\left(x_3\right)=\phi(1.32588)=1.32494$

$x_5=\phi\left(x_4\right)=\phi(1.32494)=1.32476$

Approximate root of the equation $x^3-x-1=0$ using Iteration mehtod is 1.32476

n	x_0	$x_1=\phi\left(x_0\right)$	Update	Difference x_1-x_0
2	1.5	1.35721	$x_0=x_1$	0.14279
3	1.35721	1.33086	$x_0=x_1$	0.02635
4	1.33086	1.32588	$x_0=x_1$	0.00498
5	1.32588	1.32494	$x_0=x_1$	0.00094
6	1.32494	1.32476	$x_0=x_1$	0.00018

Method-2
Let $f(x)=x^3-x-1$

$x^3-x-1=0$

$\therefore x^3-x=1$

$\therefore x\left(x^2-1\right)=1$

$\therefore x=1x^2-1$

$\therefore \phi(x)=1x^2-1$

Here

x	0	1	2
$f(x)$	-1	-1	5

Here $f(1)=-1<0$ and $f(2)=5>0$

$\therefore$ Root lies between 1 and 2

$x_0=1+22=1.5$

$x_1=\phi\left(x_0\right)=\phi(1.5)=0.8$

$x_2=\phi\left(x_1\right)=\phi(0.8)=-2.77778$

$x_3=\phi\left(x_2\right)=\phi(-2.77778)=0.1489$

$x_4=\phi\left(x_3\right)=\phi(0.1489)=-1.02267$

$x_5=\phi\left(x_4\right)=\phi(-1.02267)=21.80546$

$x_6=\phi\left(x_5\right)=\phi(21.80546)=0.00211$

$x_7=\phi\left(x_6\right)=\phi(0.00211)=-1$

$x_8=\phi\left(x_7\right)=\phi(-1)=112564.01859$

$x_9=\phi\left(x_8\right)=\phi(112564.01859)=0$

$x_{10}=\phi\left(x_9\right)=\phi(0)=-1$

$x_{11}=\phi\left(x_{10}\right)=\phi(-1)=1$

$x_{12}=\phi\left(x_{11}\right)=\phi(1)=1$

Approximate root of the equation $x_3-x-1=0$ using Iteration mehtod is 1

n	x_0	$x_1=\phi\left(x_0\right)$	Update	Difference x_1-x_0
2	1.5	0.8	$x_0=x_1$	0.7
3	0.8	-2.77778	$x_0=x_1$	3.57778
4	-2.77778	0.1489	$x_0=x_1$	2.92667
5	0.1489	-1.02267	$x_0=x_1$	1.17157
6	-1.02267	21.80546	$x_0=x_1$	22.82814
7	21.80546	0.00211	$x_0=x_1$	21.80335
8	0.00211	-1	$x_0=x_1$	1.00211
9	-1	112564.01859	$x_0=x_1$	112565.0186
10	112564.01859	0	$x_0=x_1$	112564.01859
11	0	-1	$x_0=x_1$	1
12	-1	1	$x_0=x_1$	2
13	1	1	$x_0=x_1$	0