False Position Method (RegulaFalsi Method)

Algorithm

False Position method (regula falsi method) Steps (Rule)	
Step-1:	Find points x_0 and x_1 such that $x_0<x_1$ and $f(x_0)\cdot f(x_1)<0$.
Step-2:	Take the interval $[x_0,x_1]$ and find next value $x_2=x_0-f(x_0)\cdot x_1-x_0 f(x_1)-f(x_0)$
Step-3:	If $f(x_2)=0$ then x_2 is an exact root, else if $f(x_0)\cdot f(x_2)<0$ then $x_1=x_2$, else if $f(x_2)\cdot f(x_1)<0$ then $x_0=x_2$.
Step-4:	Repeat steps 2 & 3 until $f(x_i)=0$ or $\lvert f(x_i)\rvert \leq \text{Accuracy}$

Example-1
Find a root of an equation $f(x)=x^3-x-1$ using False Position method

Solution:
Here $x^3-x-1=0$

Let $f(x)=x^3-x-1$

Here

x	0	1	2
$f(x)$	-1	-1	5

1_{st} iteration :

Here $f(1)=-1<0$ and $f(2)=5>0$

∴ Now, Root lies between $x_0=1$ and $x_1=2$

$x_2=x_0-f(x_0)\cdot x_1-x_0 f(x_1)-f(x_0)$

$x_2=1-(-1)\cdot 2-15-(-1)$

$x_2=1.16667$

$f(x_2)=f(1.16667)=-0.5787<0$

2_{nd} iteration :

Here $f(1.16667)=-0.5787<0$ and $f(2)=5>0$

∴ Now, Root lies between $x_0=1.16667$ and $x_1=2$

$x_3 = x_0 - f\left(x_0\right) \cdot x_1 - x_0 f\left(x_1\right) - f\left(x_0\right)$

$x_3 = 1.16667 - (-0.5787) \cdot 2 - 1.166675 - (-0.5787)$

$x_3 = 1.25311$

$f\left(x_3\right) = f(1.25311) = -0.28536 < 0$

3_{rd} iteration :

Here $f(1.25311) = -0.28536 < 0$ and $f(2) = 5 > 0$

∴ Now, Root lies between $x_0 = 1.25311$ and $x_1 = 2$

$x_4 = x_0 - f\left(x_0\right) \cdot x_1 - x_0 f\left(x_1\right) - f\left(x_0\right)$

$x_4 = 1.25311 - (-0.28536) \cdot 2 - 1.253115 - (-0.28536)$

$x_4 = 1.29344$

$f\left(x_4\right) = f(1.29344) = -0.12954 < 0$

4_{th} iteration :

Here $f(1.29344) = -0.12954 < 0$ and $f(2) = 5 > 0$

∴ Now, Root lies between $x_0 = 1.29344$ and $x_1 = 2$

$x_5 = x_0 - f\left(x_0\right) \cdot x_1 - x_0 f\left(x_1\right) - f\left(x_0\right)$

$x_5 = 1.29344 - (-0.12954) \cdot 2 - 1.293445 - (-0.12954)$

$x_5 = 1.31128$

$f\left(x_5\right) = f(1.31128) = -0.05659 < 0$

5_{th} iteration :

Here $f(1.31128) = -0.05659 < 0$ and $f(2) = 5 > 0$

∴ Now, Root lies between $x_0 = 1.31128$ and $x_1 = 2$

$x_6 = x_0 - f\left(x_0\right) \cdot x_1 - x_0 f\left(x_1\right) - f\left(x_0\right)$

$x_6 = 1.31128 - (-0.05659) \cdot 2 - 1.311285 - (-0.05659)$

$x_6 = 1.31899$

$f\left(x_6\right) = f(1.31899) = -0.0243 < 0$

6_{th} iteration :

Here $f(1.31899) = -0.0243 < 0$ and $f(2) = 5 > 0$

∴ Now, Root lies between $x_0 = 1.31899$ and $x_1 = 2$

$x_7=x_0-f\left(x_0\right)\cdot x_1-x_0 f\left(x_1\right)-f\left(x_0\right)$

$x_7=1.31899-(-0.0243)\cdot 2-1.318995-(-0.0243)$

$x_7=1.32228$

$f\left(x_7\right)=f(1.32228)=-0.01036<0$

7_{th} iteration :

Here $f(1.32228)=-0.01036<0$ and $f(2)=5>0$

$\therefore$ Now, Root lies between $x_0=1.32228$ and $x_1=2$

$x_8=x_0-f\left(x_0\right)\cdot x_1-x_0 f\left(x_1\right)-f\left(x_0\right)$

$x_8=1.32228-(-0.01036)\cdot 2-1.322285-(-0.01036)$

$x_8=1.32368$

$f\left(x_8\right)=f(1.32368)=-0.0044<0$

8_{th} iteration :

Here $f(1.32368)=-0.0044<0$ and $f(2)=5>0$

$\therefore$ Now, Root lies between $x_0=1.32368$ and $x_1=2$

$x_9=x_0-f\left(x_0\right)\cdot x_1-x_0 f\left(x_1\right)-f\left(x_0\right)$

$x_9=1.32368-(-0.0044)\cdot 2-1.323685-(-0.0044)$

$x_9=1.32428$

$f\left(x_9\right)=f(1.32428)=-0.00187<0$

9_{th} iteration :

Here $f(1.32428)=-0.00187<0$ and $f(2)=5>0$

$\therefore$ Now, Root lies between $x_0=1.32428$ and $x_1=2$

$x_{10}=x_0-f\left(x_0\right)\cdot x_1-x_0 f\left(x_1\right)-f\left(x_0\right)$

$x_{10}=1.32428-(-0.00187)\cdot 2-1.324285-(-0.00187)$

$x_{10}=1.32453$

$f\left(x_{10}\right)=f(1.32453)=-0.00079<0$

10_{th} iteration :

Here $f(1.32453)=-0.00079<0$ and $f(2)=5>0$

$\therefore$ Now, Root lies between $x_0=1.32453$ and $x_1=2$

$x_{11}=x_0-f\left(x_0\right)\cdot x_1-x_0 f\left(x_1\right)-f\left(x_0\right)$

$x_{11}=1.32453-(-0.00079)\cdot 2-1.324535-(-0.00079)$

$x_{11}=1.32464$

$f\left(x_{11}\right)=f(1.32464)=-0.00034<0$

Approximate root of the equation $x^3-x-1=0$ using False Position mehtod is 1.32464

n	x_0	$f\left(x_0\right)$	x_1	$f\left(x_1\right)$	x_2	$f\left(x_2\right)$	**Update**
1	1	-1	2	5	1.16667	-0.5787	$x_0=x_2$
2	1.16667	-0.5787	2	5	1.25311	-0.28536	$x_0=x_2$
3	1.25311	-0.28536	2	5	1.29344	-0.12954	$x_0=x_2$
4	1.29344	-0.12954	2	5	1.31128	-0.05659	$x_0=x_2$
5	1.31128	-0.05659	2	5	1.31899	-0.0243	$x_0=x_2$
6	1.31899	-0.0243	2	5	1.32228	-0.01036	$x_0=x_2$
7	1.32228	-0.01036	2	5	1.32368	-0.0044	$x_0=x_2$
8	1.32368	-0.0044	2	5	1.32428	-0.00187	$x_0=x_2$
9	1.32428	-0.00187	2	5	1.32453	-0.00079	$x_0=x_2$
10	1.32453	-0.00079	2	5	1.32464	-0.00034	$x_0=x_2$