Bisection Method

Algorithm

Bisection method Steps (Rule)	
Step-1:	Find points a and b such that $a<b$ and $f(a)\cdot f(b)<0$.
Step-2:	Take the interval $[a,b]$ and find next value $x_0=\frac{a+b}{2}$
Step-3:	If $f(x_0)=0$ then x_0 is an exact root, else if $f(a)\cdot f(x_0)<0$ then $b=x_0$, else if $f(x_0)\cdot f(b)<0$ then $a=x_0$.
Step-4:	Repeat steps 2 & 3 until $f(x_i)=0$ or $\left|f(x_i)\right|\le \text{Accuracy}$

Example-1

1. Find a root of an equation $f(x)=x^3-x-1$ using Bisection method

Solution:

Here $x^3-x-1=0$

Let $f(x)=x^3-x-1$

Here

x	0	1	2
$f(x)$	-1	-1	5

1^{st} iteration :

Here $f(1)=-1<0$ and $f(2)=5>0$

∴ Now, Root lies between 1 and 2

$x_0=\frac{1+2}{2}=1.5$

$f(x_0)=f(1.5)=0.875>0$

2^{nd} iteration :

Here $f(1)=-1<0$ and $f(1.5)=0.875>0$

∴ Now, Root lies between 1 and 1.5

$x_1=\frac{1+1.5}{2}=1.25$

$f(x_1)=f(1.25)=-0.29688<0$

3^{rd} iteration :

Here $f(1.25)=-0.29688<0$ and $f(1.5)=0.875>0$

∴ Now, Root lies between 1.25 and 1.5

$x_2=1.25+1.52=1.375$

$f\left(x_2\right)=f(1.375)=0.22461>0$

4_{th} iteration :

Here $f(1.25)=-0.29688<0$ and $f(1.375)=0.22461>0$

∴ Now, Root lies between 1.25 and 1.375

$x_3=1.25+1.3752=1.3125$

$f\left(x_3\right)=f(1.3125)=-0.05151<0$

5_{th} iteration :

Here $f(1.3125)=-0.05151<0$ and $f(1.375)=0.22461>0$

∴ Now, Root lies between 1.3125 and 1.375

$x_4=1.3125+1.3752=1.34375$

$f\left(x_4\right)=f(1.34375)=0.08261>0$

6_{th} iteration :

Here $f(1.3125)=-0.05151<0$ and $f(1.34375)=0.08261>0$

∴ Now, Root lies between 1.3125 and 1.34375

$x_5=1.3125+1.343752=1.32812$

$f\left(x_5\right)=f(1.32812)=0.01458>0$

7_{th} iteration :

Here $f(1.3125)=-0.05151<0$ and $f(1.32812)=0.01458>0$

∴ Now, Root lies between 1.3125 and 1.32812

$x_6=1.3125+1.328122=1.32031$

$f\left(x_6\right)=f(1.32031)=-0.01871<0$

8_{th} iteration :

Here $f(1.32031)=-0.01871<0$ and $f(1.32812)=0.01458>0$

∴ Now, Root lies between 1.32031 and 1.32812

$x_7=1.32031+1.328122=1.32422$

$f\left(x_7\right)=f(1.32422)=-0.00213<0$

9_{th} iteration :

Here $f(1.32422)=-0.00213<0$ and $f(1.32812)=0.01458>0$

∴ Now, Root lies between 1.32422 and 1.32812

$x_8=1.32422+1.328122=1.32617$

$f(x_8)=f(1.32617)=0.00621>0$

10_{th} iteration :

Here $f(1.32422)=-0.00213<0$ and $f(1.32617)=0.00621>0$

∴ Now, Root lies between 1.32422 and 1.32617

$x_9=1.32422+1.326172=1.3252$

$f(x_9)=f(1.3252)=0.00204>0$

11_{th} iteration :

Here $f(1.32422)=-0.00213<0$ and $f(1.3252)=0.00204>0$

∴ Now, Root lies between 1.32422 and 1.3252

$x_{10}=1.32422+1.32522=1.32471$

$f(x_{10})=f(1.32471)=-0.00005<0$

Approximate root of the equation $x^3-x-1=0$ using Bisection mehtod is 1.32471

n	a	$f(a)$	b	$f(b)$	$c=a+b2$	$f(c)$	Update
1	1	-1	2	5	1.5	0.875	$b=c$
2	1	-1	1.5	0.875	1.25	-0.29688	$a=c$
3	1.25	-0.29688	1.5	0.875	1.375	0.22461	$b=c$
4	1.25	-0.29688	1.375	0.22461	1.3125	-0.05151	$a=c$
5	1.3125	-0.05151	1.375	0.22461	1.34375	0.08261	$b=c$
6	1.3125	-0.05151	1.34375	0.08261	1.32812	0.01458	$b=c$
7	1.3125	-0.05151	1.32812	0.01458	1.32031	-0.01871	$a=c$
8	1.32031	-0.01871	1.32812	0.01458	1.32422	-0.00213	$a=c$
9	1.32422	-0.00213	1.32812	0.01458	1.32617	0.00621	$b=c$
10	1.32422	-0.00213	1.32617	0.00621	1.3252	0.00204	$b=c$
11	1.32422	-0.00213	1.3252	0.00204	1.32471	-0.00005	$a=c$