

M.A./M.Sc.- MATHEMATICS

Approved by

Board of Studies
Department of Mathematics
Faculty of Science and Technology
Mahatma Gandhi Kashi Vidyapith
Varanasi



Program objective of M. A. /M. Sc. (Mathematics)

The main objective of this program is to cultivate mathematical aptitude and personal skills leading to exciting careers or further study.

Programme outcomes of M.A. / M.Sc. (Mathematics)

Through our MA/MSc programmes students will develop mathematical and personal skills leading to exciting careers or further study.

Program Specific Outcomes of M.A. / M.Sc. (Mathematics)

At the end of the program, the student will be able to:

PSO1: apply the knowledge of mathematical concepts in interdisciplinary fields.

PSO2: understand the nature of abstract mathematics and explore the concepts in further details.

PSO3: model the real-world problems in to mathematical equations and draw the inferences by finding appropriate solutions.

PSO4: identify challenging problems in mathematics and find appropriate solutions.

PSO5: pursue research in challenging areas of pure/applied mathematics.

PSO6: qualify national level tests like NET/GATE etc.

Semester	Mathematics Compulsory Papers (MC)-5 credits/ papers	Mathematics Elective Papers (ME) 5 credits/ papers	Mathematics Minor Elective Paper (MME)	Research Project (4 credits)	Credits
I	MC101: Group Theory	----	MME1: Elementary Number Theory (4 credits)	1	5*4+4+4=28
	MC102: Real Analysis				
	MC103: Complex Analysis				
	MC104: Hydrodynamics				
II	MC201: Ring & Field Theory	Any one of the following: ME204: Classical Mechanics ME205: Operations Research-I	--	1	5*4+4=24
	MC202: Topology				
	MC203: Differential Equations				
III	MC301: Measure and Integration	Any two of the following: ME303: Mathematical Methods ME304: Differential Geometry ME305: Operations Research-II ME306: Discrete Mathematics	---	1	5*4+4=24
	MC302: Advanced Linear Algebra				
IV		Any four of the following: ME401: Functional analysis ME402: Normed Linear spaces and theory of Integration ME403: Algebraic Topology ME404: Fluid Mechanics ME405: Module Theory ME406: Special theory of relativity ME407: Representation theory of finite groups	-- --- --- ---	1	5*4+4=24

SEMESTER- I

MC101 :

GROUP THEORY

Credits:5

Course Objectives: The course aims to introduce the learner to the concepts of automorphism of groups, Sylow's Subgroups, normal series, composition series and Zassenhaus lemma. A study of solvable groups, nilpotent groups and commutator subgroup will be conducted.

Course Learning Outcomes: After doing this course student will be able to

CO1. compute the automorphism of groups, group action and also to prove Burnside basis theorem.

CO2. . prove Cauchy's theorem for finite groups and understand the structure of groups of order pq , p^2q and pqr .

CO3. prove Schreier's refinement theorem and Jordan-Hölder theorem and also able to compute commutator subgroups of groups.

CO4. understand the notion of solvability and nilpotency, their relationships and equivalent characterization of nilpotent groups.

Contents:

UNIT I: Automorphisms and inner automorphisms, Computation of automorphism of $\mathbf{Z}_n$ and U_n , Class equation, Action of a group G on a set, Stabilizer subgroups and orbit decomposition, Stabilizer-orbit theorem, Class equation of an action, Transitive and effective actions, Equivalence of actions, Core of a subgroup.

UNIT II: Cauchy's theorem for finite abelian groups, Cauchy's theorem for finite groups, p -groups, Sylow p -subgroups, Sylow's Theorem- I, II and III, Examples and applications, Groups of order pq , p^2q and pqr External and Internal Direct product of groups.

UNIT III: Subnormal and normal series, Zassenhaus lemma (*without proof*), Schreier's refinement theorem, Composition series, Jordan-Hölder's theorem, Chain conditions, Commutator subgroups and commutator series of a group.

UNIT IV: Solvable groups and its Examples, Insolubility of S_n for $n \geq 5$, Solvability of subgroups, factor groups and of finite p -groups, Lower and upper central series, Nilpotent groups and their equivalent characterizations.

Mappings: Unit-I implies CO1, Unit-II implies CO2, Unit-III implies CO3, Unit-IV implies CO4.

Books Recommended:

1. Ramji Lal, Algebra-I (Groups, Rings, Fields and Arithmetic), Springer Nature, 2018.
2. J. B. Fraleigh, A first Course in Abstract Algebra, Pearson Edu. Inc., 2002.
3. I. N. Herstein, Topics in Algebra, Wiley Eastern, 1975.

4. P. B. Bhattacharya, S. K. Jain and S. R. Nagpaul, *Basic Abstract Algebra (2nd Edition)*, Cambridge University Press, 1997.
5. David S Dummit and Richard M. Foote, *Abstract Algebra*, John Wiley and Sons.

MC102:

Real Analysis

Credits:5

Course Objectives: The course aims to Introduce the concepts of series of arbitrary terms, Multiplication of series, Function of several variables and Jacobians, R-S Integral and their Properties and pointwise and uniform convergence of sequence and series of functions.

Course Learning Outcomes: After studying this course the student will be able to

CO1. understand the summation of positive and negative terms of real number and application of Riemann's theorem.

CO2. understand higher order derivatives and be able to apply Taylor's theorem with remainder.

CO3. learn the concepts of integration, Existence of R-S Integral and fundamental theorem of Integral calculus.

CO4. Learn the concept of integration of a bounded function over the monotonic function.

CO5. learn concepts of convergence of sequence of functions of real numbers and the role of Weierstrass approximation theorem.

Contents:

UNIT I: Series of arbitrary terms, Convergence, divergence and oscillation, Absolute Convergence, Abel's and Dirichlet's tests, Multiplication of series, Rearrangements of terms of a series, Riemann's theorem and sum of series.

UNIT II: Functions of several variables: continuity, partial derivatives, differentiability, derivatives of functions in an open set of $\mathbb{R}^n$ into $\mathbb{R}^n$ as a linear transformations, chain rule, Taylor's theorem, inverse function theorem, implicit function theorem and explicit function theorem, Jacobians.

UNIT III: Definition and existence of Riemann-Stieltjes integral, Conditions for R-S integrability, Properties of the R-S integral, R-S integrability of functions of a function, Integration and differentiation, Fundamental theorem of Calculus.

UNIT IV: Point-wise and uniform convergence, Uniform convergence of sequences and series of functions, Cauchy's criterion for uniform convergence, uniform convergence and continuity, uniform convergence and Riemann-Stieltjes integration, uniform convergence and differentiation, Weierstrass approximation theorem,

Mappings: Unit-I implies CO1, Unit-II implies CO2, Unit-III implies CO3, Unit-IV implies CO4.

Books Recommended:

1. T. M. Apostol, *Mathematical Analysis*, Narosa Publishing House, New Delhi, 1985.

2. Walter Rudin, *Principle of Mathematical Analysis* (3rd edition) McGraw-Hill, 1976, International Student Edition.
3. S.C. Malik, *Mathematical Analysis*, Wiley Eastern, New Delhi, 1984.
4. R G Bartle, *The Elements of Real Analysis*, John Wiley & Sons

MC103:

COMPLEX ANALYSIS

Credits: 5

Course Objectives: The course aims to familiarize the learner with complex function theory, singularities and provide a glimpse of maximum principle and Schwarz' lemma. Also, study the factorization of entire functions.

Course Learning Outcomes: After studying this course the student will be able to

CO1. understand analytic function as a mapping on the plane, Mobius transformation and branch of logarithm.

CO2. know about the Maximum modulus theorem and its applications

CO3. computation of number of zeros and singularities leading to the argument principle and Rouché's theorem

CO4. know the infinite product of complex numbers and its convergence and factorization of entire functions.

Contents:

Unit I: Review of analytic functions, Branch of logarithm, Conformal mappings, Mobius transformations, Group structure of mobius transformations, Fixed points and mobius map, Cross ratios and its invariance property, Conformal mappings of half planes onto disk, automorphism of unit disk and upper half plane.

Unit II: Zeros of analytic functions, Maximum modulus principle and theorem, Minimum modulus theorem, Phragmén–Lindelöf theorem, Hadamard's three lines theorem, Hadamard's three circles theorem, Schwarz' lemma and its applications, Borel-Caratheodory theorem, Schwarz's pick lemma, Fundamental theorem of Algebra.

Unit III: Classification of Singularities, Riemann removable singularities theorem, Laurent's series and its further illustrations for singularities, isolated singularities at infinity, meromorphic functions, Argument principle, Rouché's theorem, Essential singularities, Casoriti-Weierstrass theorem, Picard's little theorem.

Unit IV: Schwarz' Reflection principle, Mittag-Leffler's theorem, Infinite product of complex numbers and its convergence, Infinite product of analytic functions, **Factorizations of entire functions:** Order of and genus of entire functions, Weierstrass factorization theorem, Hadamard's factorization theorem, Gamma function.

Mappings: Unit-I implies CO1, Unit-II implies CO2, Unit-III implies CO3, Unit-IV implies CO4.

Recommended Books:

1. Brown J. and Churchill R., Complex variables and Applications, (8th Edition).
2. L.V. Ahlfors, *Complex Analysis*, Mc Graw Hill Co., Indian Edition, 2017.
3. J. Bak and D. J. Newman, Complex Analysis, Springer.
4. S. Ponnusamy, *Foundation of complex analysis*, Narosa publication, 2003.
5. D.C. Ullrich, *Complex Made Simple*, American Mathematical Society, 2008.

MC104:

Hydrodynamics

Credits: 5

Course Objectives: The main objective of this course is to prepare a foundation for understanding the motion of fluid and develop concepts, models and techniques which enables to solve the problems of fluid flow and help in advanced studies and research in the broad area of fluid motion.

Course Learning Outcomes: After studying this course the student will be able to

CO1. understand the concept of fluid and their classification, models and approaches to study the fluid flow.

CO2. formulate mass and momentum conservation principle and obtain solution for nonviscous flow.

CO3. know potential theorems, minimum energy theorem and circulation theorem.

CO4. understand two dimensional motion, circle theorem and Blasius theorem.

CO5. Understand motion of sphere through a liquid at rest at infinity and Equation of motion of a sphere.

Contents:

UNIT I: Methods of describing fluid motion and their relationship, Equation of continuity by Euler's method in vector form, cartesian, cylindrical and polar coordinates, Equation of continuity by Lagrangian method, some symmetrical forms of equation of continuity, Boundary surfaces.

UNIT II: Streamlines, Path lines, Velocity potential, Vorticity vector and Vortex lines, Irrotational and rotational motions, Euler's Equation of motion in vector form, Euler's Equation of motion in Cartesian, cylindrical, and spherical polar coordinates, Impulsive forces, Equation of motion under impulsive forces.

UNIT III: Bernoulli's equation, Bernoulli's theorem and its applications, Motion in two-dimensions, Stream functions and its physical significance, Complex potentials, Sources and sinks, doublets, complex potential due to a doublet in 2-D, Images, Image of a source and a doublet with respect to a line.

UNIT IV: Image of a doublet with respect to a circle, Milne-Thompson Circle Theorem. Theorem of Blasius, Motion of a sphere through a liquid at rest at infinity, Liquid streaming past a fixed sphere, Equation of motion of a sphere, Concentric Spheres.

Mappings: Unit-I implies CO1, Unit-II implies CO2, Unit-III implies CO3, Unit-IV implies CO4.

Recommended Books:

1. M. D. Raisinghania, Fluid Dynamics, S. Chand New Delhi, 2006 (7th Edition).
2. B. D. Sharma and A. K. Sachdeva, Elementary Hydrodynamics, , Kedarnath Ramnath, Meerut.
3. W. H. Besant and A. S. Ramsey, A Treatise on Hydrodynamics, CBS Pub. Delhi, 1988.
4. S. W. Yuan, Foundations of Fluid Dynamics, Prentice-Hall of India, 1988.

Mathematics Minor Elective Paper

MME1:	ELEMENTARY NUMBER THEORY	Credits:4
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Course Objective: The objectives of this course is to study the basic concepts of number theory, Diophantine equations, Goldbach conjecture and primes of the form $4n + 1$ and $4n + 3$, Important theorems like Fermat's, Euler's and Wilson Theorems, different arithmetic functions, Quadratic Reciprocity and some symbols representing them etc.

Course Outcomes: Upon successful completion of this course,

CO1: The students would have knowledge and skills to solve problems in elementary number theory and also apply elementary number theory to cryptography.

CO2: The students will understand the concepts of, binary and decimal representations of integers, Solving linear congruences, Chinese Remainder theorem and proving Fermat's and Wilson theorem.

CO3: The students would have knowledge about different types of prime numbers, fermat's and Wilson's theorem, computation of no. of divisors and sum of divisors, Mobius Inversion Formula, Euler's phi function, Euler's Theorem, Unit Group of $\mathbb{Z}_n$ and how to compute their primitives.

CO4: The students would have knowledge about, why we need Quadratic reciprocity, Legendre and Jacobi Symbols, Solving Quadratic Congruences, Perfect Numbers, Carmichael Numbers, Fibonacci Numbers.

Contents:

Unit 1- Well Ordering Principle, Archimedean Property, First Principle of Finite Induction, Binomial Theorem, Division Algorithm, Greatest common divisor (GCD), Euclid's lemma, Euclidean Algorithm, least common multiple(LCM), The Diophantine Equation of the form $ax + by = c$.

Unit II- Prime and Composite numbers, Fundamental Theorem of Arithmetic, Pythagoras theorem, Euclid's theorem for number of primes, Goldbach conjecture, Number of primes of the form $4n+1$ and $4n+3$, Congruence modulo n and their properties, Binary and decimal representation of integers, divisibility rule for 9 and 11, Linear congruences and Chinese Remainder Theorem.

Unit III- Pseudo Primes, Mersenne Primes, Fermat's Primes, Fermat's Little theorem, Wilson's theorem. The functions τ and σ and their properties, Mobius Inversion Formula, Euler's phi function and their properties, Euler's Theorem, Unit Group of $\mathbb{Z}_n$ and their primitives.

Unit IV- Quadratic Reciprocity, Euler's Criterion, Legendre Symbol and its properties, Jacobi Symbol and its properties, Gauss's Lemma, Quadratic reciprocity law, Quadratic congruences with composite moduli, Perfect Numbers, Carmichael Numbers, Fibonacci Numbers.

Mappings: Unit-I implies CO1, Unit-II implies CO2, Unit-III implies CO3, Unit-IV implies CO4.

Books Recommended:

1. David M. Burton, Elementary Number Theory, Seventh Edition, McGraw Hill.
2. G.A. Jones and J.M. Jones, Elementary Number Theory, Springer-Verlag, 1998.
3. W. Sierpinski, Elementary Theory of Numbers, North-Holland, Ireland, 1988.
4. Niven, S.H. Zuckerman and L.H. Montgomery, An Introduction to the Theory of Numbers, John Wiley, 1991.
5. Joseph H. Silverman, A Friendly Introduction to Number Theory, 4th ed., Pearson.
6. Thomas Koshy, Elementary Number Theory with Applications, 2nd ed., Academic Press.

SEMESTER-II

MC201:

Ring & Field Theory

Credits: 5

Course Objectives: The main objective of this course to introduce algebraic structures and its properties which plays crucial roles in real world problems. In this course we study ring of polynomials over a field in details. Also we study fields in detail with a focus on Galois theory which provides a link between group theory and roots of polynomials.

Course Learning Outcomes: After studying this course the student will be able to

CO1. identify and construct examples of fields, distinguish between Maximal and prime ideals.

CO2. To find the relationship between UFD, PID, ED and check the irreducibility criteria for polynomials.

CO3. classify finite fields using roots of unity and Galois theory and prove that every finite separable extension is simple.

CO4. use Galois theory of equations to prove that a polynomial equation over a field of characteristic zero is solvable by radicals iff its group (Galois) is a solvable group and hence deduce that a general quintic equation is not solvable by radicals.

Contents:

UNIT I: Elementary Ring theory: Homomorphisms and isomorphisms of a ring, Embedding of rings, Quotient fields, Subrings, ideals and isomorphism theorems, quotient rings, second isomorphism theorems of rings, Maximal ideals and prime ideals.

UNIT II: Polynomial rings: Polynomial rings over Integral domains, Division Algorithm, remainder and factor theorem, **Arithmetic in Rings:** Division in rings, Prime and irreducible elements, G.C.D and L.C.M., Principal ideal domains, Euclidean domains, Chinese remainder theorem in rings, Unique factorization domains (U.F.D).

UNIT III: Arithmetic in Rings (continued): Gauss lemma, irreducibility criterion over U.F.D., Eisenstein's irreducibility criterion. **Field Theory:** Finite fields, Extension fields, Perfect fields, Finite extension, Simple, algebraic and transcendental extensions, Primitive elements, Simple extensions, Factorization of polynomial in extension fields, Splitting fields, Algebraically closed fields and algebraic closure (definition only).

UNIT IV: Galois Theory: Galois Group, Fixed field, Galois extensions, Computation of Galois Groups, Dedekind theorem, Properties of Galois groups, Fundamental theorem of Galois theory, Geometric constructions, Galois theory of equation.

Mappings: Unit-I implies CO1, Unit-II implies CO2, Unit-III implies CO3, Unit-IV implies CO4.

Books Recommended:

1. D. S. Dummit and R. M. Foote, Abstract Algebra, John Wiley & Sons, N.Y., 2003.
2. N. S. Gopalakrishnan, University Algebra, Wiley Eastern, New Delhi, 1986.
3. R Lal, Algebra-II, Springer Verlag, 2018.
4. J. B. Fraleigh, A first Course in Abstract Algebra, Pearson Edu. Inc., 2002.

MC202:**TOPOLOGY****Credits: 5**

Course Objectives: To introduce basic concepts of point set topology, basis and subbasis for a topology and order topology. Further, to study continuity, homeomorphisms, open and closed maps, product and box topologies and introduce notions of connectedness, path connectedness, local connectedness, local path connectedness, convergence, nets, countability axioms and compactness of spaces.

Course Learning Outcomes: After studying this course the student will be able to

CO1. determine interior, closure, boundary, limit points of subsets and basis and subbasis of topological spaces.

CO2. check whether a collection of subsets is a basis for a given topological spaces or not, and determine the topology generated by a given basis.

CO3. identify the continuous maps between two spaces and maps from a space into product space and determine common topological property of given two spaces.

CO4. determine the connectedness and path connectedness of the product of an arbitrary family of spaces.

CO5. find Hausdorff spaces using the concept of net in topological spaces and learn about 1st and 2nd countable spaces, separable and Lindelöf spaces.

CO6. learn Bolzano-Weierstrass property of a space and prove Urysohn's lemma and Tietze extension theorem.

Contents:

Unit I: Topological spaces, Definition and examples, Open and Closed sets, Interior, closure, boundary and limit points of subsets.

Unit II. Continuity, Homeomorphism, Uniform Continuity, Separated sets, Connected and disconnected sets,

Unit III: Continuity and connectedness, components, totally disconnected space, locally connected space, compactness, countable, Sequential and local compactness, compactness in metric space, continuity and compactness .

Unit IV: First and Second countable spaces, Separable and Lindelöf spaces, The separation axioms and their characterizations and basic properties Urysohn's lemma, Tietze extension theorem.

Mappings: Unit-I implies CO1, Unit-II implies CO2, Unit-III implies CO3, Unit-IV implies CO4.

Books Recommended:

1. James R. Munkres, *Topology*, 2nd Edition, Pearson International, 2015.
2. J. Dugundji, *Topology*, Prentice-Hall of India, 1966.
3. G.E. Bredon, *Topology and Geometry*, Springer, 2014.
4. J.L. Kelley, *General Topology*, Dover Publications, 2017.
5. T.B. Singh, *Elements of Topology*, CRC Press, Taylor & Francis, 2013.

MC203:

DIFFERENTIAL EQUATIONS

Credits: 5

Course Objectives: The objective of this course is to study the solutions of first order ODE's, linear second order ODE's, boundary value problems, eigenvalues and eigen functions of Sturm Liouville systems, stability of systems of ODEs and PDEs.

Course Learning Outcomes: After studying this course the student will be able to

CO1. know about existence, uniqueness and solutions of first order ODE's, properties of zeros of solutions of linear second order ODE's, boundary value problems.

CO2. understand with eigenvalues and eigenfunctions of Sturm–Liouville systems, and the solutions of initial and boundary value problems.

CO3. be well equipped to undertake any advanced course on ordinary as well as partial differential equations.

Contents:

Unit I: Picard's method of successive approximation for initial value problems, Problems of Existence and uniqueness of initial value problems $dy/dx = f(x,y)$, $y(x_0) = y_0$, Lipschitz condition, Picard's theorem, Wronskian.

Unit II: Homogeneous linear partial differential equations with constant coefficients, Complementary functions and particular integrals, Non-homogeneous linear partial differential equations with constant coefficients, Reducible and irreducible linear operators, Solution to homogeneous and non-homogeneous linear partial differential equations with constant coefficients.

Unit III: Partial differential equations reducible to equations with constant coefficients, Classification of partial differential equation of second order, Reduction to canonical forms.

Unit IV: Orthogonality, Orthogonal and orthonormal set of functions, Orthogonal and orthonormal set of functions w.r.t. Weight function, Gram-Schmidt process of orthogonalization, Sturm-Liouville (S-L) problems: eigenvalues and eigenfunctions, reality of eigenvalues, orthogonality of Legendre polynomials and Bessel's functions.

Mappings: Unit-I implies CO1, Unit-II implies CO2, Unit-III implies CO3, Unit-IV implies CO4.

Books Recommended:

1. E.A. Coddington, *An Introduction to Ordinary Differential Equations*, Dover Publications, 2012.

2. T. Myint-U, *Ordinary Differential Equations*, Elsevier, North-Holland, 1978.
3. S.L. Ross, *Differential Equations*, Second Edition, John Wiley & Sons, India, 2007.
4. I.N. Sneddon, *Elements of Partial Differential Equations*, Dover Publications, 2006.
5. M. D. Raisinghania, *Advanced Differential Equations*, S. Chand, New Delhi, reprints, 2019.

Elective papers (Any one of the following)

ME204:

Classical Mechanics

Credits: 5

Course Objectives: The purpose of this course is to concepts of momentum energy of rigid body generalized coordinates, Lagrange's and Hamilton's equation of motion of rigid body and variational principals and canonical transformations.

Course Learning Outcomes: After successful completion of this course students are able

CO1. to know energy, momentum of rigid body rotating about fixed point.

CO2. to know some well known problems and their equations of motions.

CO3. to know Lagrange's equations of motions for constrained motion under finite forces and motion under impulsive forces, the motion of small oscillations and Hamilton's equation of motion.

CO4. to know the concept of variational principles and its application and the notion of canonical transformation, phase spaces and invariance of phase space under canonical transformation.

Contents:

UNIT I: Rotation of a vector in two and three dimensional fixed frame of references. Kinetic energy and angular momentum of rigid body rotating about its fixed point, Euler dynamical and geometrical equations of motion, Generalized coordinates, momentum and force components, Lagrange equations of motion under finite forces, cyclic coordinates and conservation of energy.

UNIT II: Lagrangian approach to some known problems-motions of simple, double, spherical and cycloidal pendulums, motion of a particle in polar system, motion of a particle in a rotating plane, motion of a particle inside a paraboloid, motion of an insect crawling on a rod rotating about its one end, motion of masses hung by light strings passing over pulleys, motion of a sphere on the top of a fixed sphere and Euler dynamic equations.

UNIT III: Lagrange equations for constrained motion under finite forces. Lagrange equations of motion under impulses, motion of parallelogram about its centre and some of its particular cases, Small oscillations for longitudinal and transverse vibrations, Equations of motion in Hamiltonian approach and its applications on known problems as given above, Conservation of energy, Legendre dual transformations.

UNIT IV: Hamilton principle and principle of least action, Hamilton-Jacobi equation of motion, Hamilton-Jacobi theorem and its verification on the motions of a projectile under gravity in two dimensions and motion of a particle describing a central orbit, Phase space, canonical transformations, conditions of canonicity, cyclic relations, generating functions, invariance of elementary phase space, canonical transformations form a group and Liouville theorem. Poisson

brackets, Poisson first and second theorems, Poisson, Jacobi identity and invariance of Poisson bracket.

Mappings: Unit-I implies CO1, Unit-II implies CO2, Unit-III implies CO3, Unit-IV implies CO4.

Books Recommended:

1. A. S. Ramsay, *Dynamic* –Part II.
2. N. C. Rana and P.S. Joag, *Classical Mechanics*, Tata McGraw-Hill, 1991.
3. H. Goldstein, *Classical Mechanics*, Narosa Publishing House, New Delhi, 1990.
4. Naveen Kumar, *Generalized Motion of Rigid Body*, Narosa Publishing House, New Delhi, 2004.

ME205:

Operations Research-I

Credits: 5

Course outcomes: The aim of this course is to teach the student the application of various numerical technique for variety of problems occurring in daily life.

Course Learning Outcomes: After successful completion of this course student will be able

CO1: to know the concept of linear programming problems

CO2: to know the method for solving extremal problems and its dual problems

CO3: to understand transportation and assignment problems

CO4: to know the methodology for solving networking problems.

Contents:

UNIT I: Operations Research and its Scope, Necessity of Operations Research in Industry, Linear Programming – Simplex Method.

UNIT II: Duality in Linear Programming, Dual Simplex Method, Parametric Linear Programming.

UNIT III: Transportation and Assignment Problems

UNIT IV: Integer programming, Inventory Control-I.

Mappings: Unit-I implies CO1, Unit-II implies CO2, Unit-III implies CO3, Unit-IV implies CO4.

Books Recommended:

1. H. A. Taha. *Operations Research – An Introduction*, Macmillan Publishing Co., Inc., New York.
2. S. S. Rao, *Optimization Theory and Applications*, Wiley Eastern Ltd., New Delhi.
3. Kanti Swarup, P. K. Gupta and Man Mohan, *Operations Research*, Sultan Chand & Sons, New Delhi.

SEMESTER-III

MC301:

Measure and Integration

Credits: 5

Course Objectives: The main objective is to familiarize with the Lebesgue outer measure, Measurable sets, Measurable functions, Integration, Convergence of sequences of functions and their integrals, Functions of bounded variation.

Course Learning Outcomes: After studying this course the student will be able to

CO1. verify whether a given subset of real valued function is measurable.

CO2. understand the requirement and the concept of the Lebesgue integral (a generalization of the Riemann integration) along its properties and relationship between Lebesgue and Riemann integral.

CO3. know about the concepts of functions of bounded variations and the absolute continuity of functions with their relations. Extend the concept of outer measure in an abstract space and integration with respect to a measure.

CO4. learn and apply Holder and Minkowski inequalities in p -spaces and understand completeness of spaces and convergence in measures.

Contents:

UNIT I: Semi-algebras, algebras, monotone class, σ -algebras, measure and outer measures, Caratheodory extension process of extending a measure on a semi-algebra to generated σ -algebra, completion of a measure space.

UNIT II: Borel sets, Lebesgue outer measure and Lebesgue measure on $\mathbb{R}$, translation invariance of Lebesgue measure, existence of a non-measurable set, characterizations of Lebesgue measurable sets, the Cantor's set, Non-measurable sets.

UNIT III: Measurable functions on a measure space and their properties, Borel and Lebesgue measurable functions, simple functions and their integrals, Lebesgue integral on $\mathbb{R}$ and its properties, Riemann and Lebesgue integrals.

UNIT IV: Bounded convergence theorem, Fatou's lemma, Lebesgue monotone convergence theorem, Lebesgue dominated convergence theorem.

Mappings: Unit-I implies CO1, Unit-II implies CO2, Unit-III implies CO3, Unit-IV implies CO4.

Recommended Books:

1. G. de Barra, *Measure Theory and Integration*, New Age International Ltd., New Delhi, 2014.
2. M. Capinski and P.E. Kopp, *Measure, Integral and Probability*, Springer, 2005.
3. E. Hewitt and K. Stromberg, *Real and Abstract Analysis: A Modern Treatment of the Theory of Functions of a Real Variable*, Springer, Berlin, 1975.

4. H.L. Royden and P.M. Fitzpatrick, *Real Analysis*, Fourth Edition, Pearson, 2015.
5. P. K. Jain, *Measure and Integration*, New Age International (P) Ltd., New Delhi.

MC302:

ADVANCED LINEAR ALGEBRA

Credits:5

Course Objectives: The aim of this course is to study the concept of inner product spaces, Eigen Values, Eigen Vectors, Diagonalization and triangularisation of matrices, Reduction of matrices over polynomial rings over a field, π -modules, Reduction to Jordan canonical forms will be conducted and the students will be able to know the Taylor's formula.

Course Learning Outcomes: After doing this course student will be able

CO1. to know the geometry of general vector spaces and to prove the Cauchy Schwarz inequality, Bessel's Inequality and their applications.

CO2. To find the orthonormal basis of a subspace generated by finite number of vectors of an inner product space and find eigenvalues, eigenvectors of a given linear operator.

CO3. To prove Cayley Hamilton Theorem for a linear operator on finite dimensional space and know the notion of T-conductors and T-annihilators.

CO4. Characterise diagonalizability in terms of projections.

CO5. Understand rational and Jordan canonical forms and reductions of Jordan Canonical Forms..

UNIT I: Inner product spaces, Cauchy-Schwarz inequality, orthogonal vectors, Orthonormal basis, Bessel's inequality, Gram-Schmidt orthogonalization process, Eigenvalues, eigenvectors, and eigenspaces, Algebraic and geometric multiplicities of eigenvalues.

UNIT II: Diagonalization, Cayley-Hamilton theorem, Invariant subspaces, T-conductors and T-annihilators, Minimal polynomials of linear operators and matrices, Characterization of diagonalizability in terms of multiplicities and also in terms of the minimal polynomial, Triangulability, Simultaneous triangulation and diagonalization.

UNIT III: Reduction of matrices over polynomial rings over a field, Similarity of matrices and $F[x]$ -module structure, Projections, Invariant direct sums, Characterization of diagonalizability in terms of projections, Primary decomposition theorem.

UNIT IV: Diagonalizable and nilpotent parts of a linear operator, Rational canonical form of matrices, Elementary Jordan matrices, Reduction to Jordan canonical form, Semi-simple operators, Taylor formula.

Mappings: Unit-I implies CO1, Unit-II implies CO2, Unit-III implies CO3, Unit-IV implies CO4.

Books Recommended:

1. K. Hofmann and R. Kunze, *Linear Algebra*. Prentice Hall of India, New Delhi, 1972.

2. D. S. Dummit and R. M. Foote, Abstract Algebra, John Wiley & Sons, N.Y. 2003.
3. N. Jacobson, Basic Algebra, Vol. 1, Hindustan Publishing Co., New Delhi, 1984.
4. Vivek Sahai and Vikas Bist, Linear Algebra.
5. T. S. Blyth and Robertson, further Linear Algebra, springer.

Elective papers (Any two of the following)

ME303:

MATHEMATICAL METHODS

Credits: 5

Course Objectives: The objective of this course is to study Variational problems with fixed and moving boundaries and their solutions, one and two dimensional wave equation and to study the linear integral equations Volterra and Fredholm type .

Course Learning Outcomes: After studying this course the student will be able to

CO1. know about functionals and extremals

CO2. know Euler's equation and its application in solving Variational Problems.

CO3. to solve the linear integral equations and find its resolvent and n^{th} iterated kernels.

CO4. know the notion of heat, wave equations and their solutions

Contents:

UNIT I: Calculus of Variations: Functionals and extremals, Variation and its properties, The fundamental lemma of Calculus of Variations, Euler equations and its applications, Geodesics, Variational problems for functionals of several dependent and independent variables.

UNIT II: Functionals dependent on higher derivatives, Variational problems in Parametric forms, Simple applications, Isoperimetric problems, Reciprocity principles, Variational problems with moving (or free) boundaries, Brachistochrone problems, Variational problems with moving boundary for a functional dependent on two functions.

UNIT III: Classification of linear integral equations, Relation between differential and integral equations. Fredholm equations of second kind with separable kernels, Fredholm alternative theorem, Eigen values and eigen functions, Method of successive approximation for Fredholm and Volterra equations, Resolvent kernel.

UNIT IV: Heat and Wave equations: Introduction, Derivation of one and two dimensional wave equation, Derivation of one dimensional heat equation, Derivation of fourier equation of heat conduction, Laplace equation in cartesian, polar, spherical and cylindrical coordinates, Method of separation of variables for one dimensional heat and wave equations.

Mappings: Unit-I implies CO1, Unit-II implies CO2, Unit-III implies CO3, Unit-IV implies CO4.

Recommended Books:

1. E. Kreyszig, Advanced Engineering Mathematics, Wiley India Pvt. Ltd., 8th Edition, 2001.
2. L. Elsgolts, Differential Equations and Calculus of Variations, Mir Publishers, 1970.
3. A. S. Gupta, Calculus of Variations, Prentice Hall of India, New Delhi, 1999.
4. M. D. Raisinghania, Advanced Differential Equations, S. Chand, New Delhi, reprints, 2019.
5. M. D. Raisinghania, Integral equations and boundary value Problems, S. Chand, New Delhi, 2007.

ME304:

Differential Geometry

Credits-5

Course Objectives: The aim of this course is to study local and global theory of surfaces and fundamental equations of surface theory.

Course outcomes: After successful completion of this course student will be able

CO1: to determine and calculate curvature of curves in different coordinate systems and existence, uniqueness of curves.

CO2: Know the Local theory of surfaces.

CO3: know global theory of surfaces.

CO4: know some fundamental equations of surface theory.

Contents:

Unit I : Curves in R^2 and R^3 : Basic Definitions and Examples. Arc Length. Tangent and osculating plane, Principal normal and binormal, Curvature and torsion, The Frenet-Serret Apparatus. The Fundamental Existence and Uniqueness Theorem for Curves.

Unit II: (Local theory of Surfaces) Definition of a surface, Nature of points on a surface, Representation of a surface, Curves on surfaces, Tangent plane and surface normal, The general surfaces of revolution, Helicoids, Metric on a surface—The first fundamental form, Direction coefficients on a surface, Families of curves, Orthogonal trajectories,

Unit III: (Global Theory of Surfaces) Normal Curvature, The second fundamental Form, Meusnier Theory, classification of points on a surface, Principal curvatures, Mean Curvature and Gaussian Curvature, Umbilic points, Lines of Curvature and Dupin's Theorem. Minimal Surfaces.

Unit IV: Fundamental Equation of Surface Theory, Gauss's Equations, weingarten equations, mainardi-Codazzi equations, parallel surfaces.

Mappings: Unit-I implies CO1, Unit-II implies CO2, Unit-III implies CO3, Unit-IV implies CO4.

Books Recommended:

1. Christian Bär, Elementary Differential Geometry, Cambridge University Press, 2010.
2. O'Neill, Elementary Differential Geometry, Elsevier 2006.
4. Pressley, Elementary Differential Geometry, Springer 2010.
5. D. Somasundaram, Differential Geometry, A first course, Alpha Science International Ltd., 2005.

ME305:

OPERATION RESEARCH-II

Credits: 5

Course outcomes: The aim of this course is to teach the student the application of various numerical technique for variety of problems occurring in daily life.

Course Learning Outcomes: After successful completion of this course student will be able

CO1: to know the concept of game theory and methods of finding suitable solution.

CO2: to know the method for solving extremal problems whose objective functions/constraints are non-linear.

CO3: to take multistage problems.

CO4: to know the methodology for solving networking problems.

Contents:

Unit I: Game Theory: Two person zero sum games, Games with mixed strategies.

Unit II: Non Linear Programming: one and multi variable unconstrained optimization, Kuhn-Tucker, Condition for constrained optimization, Quadratic Programming, Separable Programming

Unit III: Dynamic Programming and sequencing problem, basic terms used in sequencing, Processing n jobs through to machine, Processing n jobs through k machine, Processing 2 jobs through k machine.

Unit IV: Network - Scheduling by PERT/CPM.

Mappings: Unit-I implies CO1, Unit-II implies CO2, Unit-III implies CO3, Unit-IV implies CO4.

Books Recommended:

1. F. S. Hiller and G. J. Lieberman, *Introduction to Operations Research* (6th Edition), McGraw-Hill International Edition, 1995.
2. G. Hadley, *Nonlinear and Dynamic Programming*, Addison Wesley.
3. H. A. Taha, *Operations Research –An Introduction*, Macmillan.
4. Kanti Swarup, P. K. Gupta and Man Mohan, *Operations Research*, Sultan Chand & Sons, New Delhi.
5. S. S. Rao, *Optimization Theory and Applications*, Wiley Eastern.
6. N. S. Kambo, *Mathematical Programming Techniques*, Affiliated East-West Press Pvt. Ltd., New Delhi.

ME306:

Discrete Mathematics

Credits: 5

Course Objectives: The course objective is to provide overview of discrete mathematics to Students will learn about topics such as logic and proofs, sets and functions, recursion, graph theory, Boolean algebra. Students will also learn its applications in computer sciences.

Course outcomes: Upon successful completion of this course, student will be able

CO1: to have the knowledge of lattices and behaviour as an algebraic system.

CO2: to know new algebraic structure, Boolean algebra, its behaviour as Lattices and its application in circuit theory.

CO3: to know the notion of graph, some special graph and coloring .

CO4: to understand searching algorithms for a trees and some other algorithms, kruskal and warshall's algorithm.

Contents:

UNIT I: Lattices: Lattices as partially ordered sets and their properties Lattices as Algebraic systems, Sublattices. Direct product, and Homomorphisms. Some Special Lattices, e.g., Complete, Complemented and Distributive Lattices.

UNIT II: Boolean Algebra: Boolean Algebras as Lattices, Various Boolean identities, Direct Product and Homomorphisms, Canonical Forms, Minimization of Boolean Functions. Application of Boolean Algebra to Switching Theory (using AND, OR & NOT gates), The Karnaugh Map method.

UNIT III: Graph Theory: Definition of (undirected) Graphs. Paths, Circuits, Cycles and Subgraphs, Induced Subgraphs, Degree of vertex, Connectivity, Planar Graphs and their properties, Tress, Euler's Formula for connected Planar Graphs. Complete & Complete Bipartite Graphs.

UNIT IV: Spanning Tress, Cut-sets, Fundamental Cut-sets and Cycles, Minimal Spanning Tress and Kruskal's Algorithm, Matrix Representation of Graphs, Euler's Theory of a Vertex, Weighted undirected Graphs, Strong Connectivity & Warshall's Algorithm, Directed Tress. Search Trees, Tree Traversals.

Mappings: Unit-I implies CO1, Unit-II implies CO2, Unit-III implies CO3, Unit-IV implies CO4.

Books Recommended:

1. D. B. West, Graph Theory, Pearson Publ., 2002.
2. F. Harary, Graph Theory, Narosa Publishing House, New Delhi.
3. R. Diestel, Graph Theory, Springer, 2000.
4. R. Garnier and J Taylor, Discrete Mathematics for New Technology, IOP Publishing, 2002.
5. A.C. Yadav, Elements of graph theory, Golden Vally Publication, Discrete mathematics and its applications.
6. K.H.Rosen, Discrete Mathematics and its applications, Mc Graw Hill.

SEMESTER IV

Elective Papers (Any four of the following)

ME401:

FUNCTIONAL ANALYSIS

Credits: 5

Course Objectives: To familiarize with the basic tools of Functional Analysis involving normed spaces, Banach spaces and Hilbert spaces, their properties and the bounded linear operators from one space to another.

Course Learning Outcomes: After studying this course the student will be able to

CO1. verify the requirements of a norm, completeness with respect to a norm, relation between compactness and dimension of a space, check boundedness of a linear operator and relate to continuity, convergence of operators by using a suitable norm, compute the dual spaces.

CO2. distinguish between Banach spaces and Hilbert spaces, decompose a Hilbert space in terms of orthogonal complements, check totality of orthonormal sets and sequences, represent a bounded linear functional in terms of inner product, classify operators into self-adjoint, unitary and normal operators.

CO3. extend a linear functional under suitable conditions, compute adjoint of operators, check reflexivity of a space, ability to apply uniform boundedness theorem, open mapping theorem and closed graph theorem, check the convergence of operators and functional and weak and strong convergence of sequences.

CO4. compute the spectrum of operators and classify the set into subclasses, show the spectrum to be nonempty, give expansion of resolvent operator.

Contents:

Unit I: Normed spaces, Banach spaces, Finite dimensional normed spaces and subspaces, Compactness and finite dimension, Bounded and continuous linear operators, Linear operators and functionals on finite dimensional spaces, Normed spaces of operators, Dual spaces.

Unit II: Hilbert spaces, Orthogonal complements and direct sums, Bessel's inequality, Total orthonormal sets and sequences, Representation of functionals on Hilbert spaces, Hilbert adjoint operators, Self-adjoint, unitary and normal operators.

Unit III: Hahn Banach theorems for real and complex normed spaces, Adjoint operator, Reflexive spaces, Uniform boundedness theorem strong and weak convergence, Convergence of sequences of operators and functional.

Unit IV: Open mapping theorem, Closed graph theorem, Spectrum of an operator, Spectral properties of bounded linear operators, Non-emptiness of the spectrum.

Mappings: Unit-I implies CO1, Unit-II implies CO2, Unit-III implies CO3, Unit-IV implies CO4.

Books Recommended:

1. G. Bachman and L. Narici, *Functional Analysis*, Dover Publications, 2000.

2. R. Bhatia, *Notes on Functional Analysis*, Hindustan Book Agency, India, 2009.
3. E. Kreyszig, *Introductory Functional Analysis with Applications*, John Wiley & Sons, India, 2006.
4. M. Schechter, *Principles of Functional Analysis*, Second Edition, AMS, 2001.
5. G. F. Simmons, *Introduction to Topology and Modern Analysis*, McGraw Hill, 1963.
6. S. Ponnusamy, *Foundations of Functional Analysis*, Narosa Publishing House, New Delhi, 2002.

ME402

Normed Linear spaces and theory of Integration

Credits: 5

Course Objectives: The purpose of this course is to Introduce negative real valued and complex valued measures, decomposition of the measure space and the measure, extension of a measure, integral representation of measures and functionals, product measure and uniqueness of Lebesgue measure in Euclidean space.

Course Learning Outcomes: After studying this course the student will be able to

CO1. understand signed measures and complex measures, ability to use Hahn decomposition, Jordan decomposition, Radon-Nikodym theorem and recognize singularity of measures.

CO2. verify conditions under which a measure defined on a semi-algebra or algebra is extendable to a sigma-algebra and to get the extended measure, and to prove the uniqueness up to multiplication by a scalar of Lebesgue measure in $\mathbb{R}^n$ as a translation invariant Borel measure.

CO3. learn and apply Riesz representation theorem for a bounded linear functional on spaces, understand product measure and the results of Fubini and Tonelli.

CO4. to understand the concepts of Baire sets, Baire measures, regularity of measures on locally compact spaces, Riesz-Markov representation theorem related to the representation of a bounded linear functional on the space of continuous functions.

Contents:

UNIT I: The L^p -space, Convex functions, Jensen's inequality, Holder's and Minkowski's inequalities, Completeness of Convergence in measure, Almost uniform convergence.

UNIT II: Signed measure, Hahn and Jordan decomposition theorems, Absolutely continuous and singular measures, Radon Nikodym theorem, Lebesgue decomposition, Riesz representation theorem, Extension theorem (Carathéodory).

UNIT III: Product measures, Tonelli's theorem, Fubini's theorem, Baire sets, Baire measure, Continuous functions with compact support.

UNIT IV: Regularity of measures on locally compact spaces, Integration of continuous functions with compact support, Riesz-Markoff theorem.

Mappings: Unit-I implies CO1, Unit-II implies CO2, Unit-III implies CO3, Unit-IV implies CO4.

Books Recommended:

- [1] C.D. Aliprantis and O. Burkinshaw, Principles of Real Analysis, Academic Press, Indian Reprint, 2011.
- [2] A.K. Berberian, Measure and Integration, AMS Chelsea Publications, 2011.
- [3] P.R. Halmos, Measure Theory, Springer, 2014.
- [4] M.E. Taylor, Measure Theory and Integration, American Mathematical Society, 2006.
- [5] H.L. Royden and P.M. Fitzpatrick, Real Analysis, Fourth Edition, Pearson, 2015.

ME403:

Algebraic Topology

Credits: 5

Course Objectives: The goal of this course is to introduce the basic notion of study of topological spaces through groups.

Course Learning Outcomes: After successful completion of this course student will be able

CO1. to know the basic notion of homotopy theory.

CO2. To familiarize the affine spaces and maps.

CO3. to compute fundamental group of , etc.

CO4. to familiarize the basic notion of homology groups.

Contents:

Unit I: Basic notions, standard n -simplex, stereographic projection, Brouwer fixed point theorem, retract, categories and functions, homotopy category, homotopy, null homotopy, identification.

Unit II: Cone, paths and path connected space, path components & some results, deformation retract, mapping cylinder of continuous function, Affine spaces and Affine maps.

Unit III: Gluing lemma, Relative homotopy, Path classes, groupoid of path classes, Fundamental group, Functor from category of pointed space into category of groups, some results, fundamental group of product space, free homotopy, fundamental group of contractible space, circle, H -spaces and its fundamental group, top groups .

Unit IV: Singular homology: Free abelian groups presentations, orientation of standard n -simplex, Singular n -simplex, n -chains, boundary, singular complex n -cycles, n -boundaries, Homology groups, Betti numbers, dimension axioms and compact support, Hurewicz theorem complexes, n th homology groups of complex, Exact homology sequences.

Mappings: Unit-I implies CO1, Unit-II implies CO2, Unit-III implies CO3, Unit-IV implies CO4.

Books Recommended:

1. G.E. Bredon, *Geometry and Topology*, Springer, 2014.
2. W.S. Massey, *A Basic Course in Algebraic Topology*, World Publishing Corporation, 2009.
3. J.J. Rotman, *An Introduction to Algebraic Topology*, Springer, 2011.
4. T.B. Singh, *Elements of Topology*, CRC Press, Taylor & Francis, 2013.
5. E.H. Spanier, *Algebraic Topology*, Springer-Verlag, 1989.
6. Hatcher, , *Algebraic Topology*

ME404:

FLUID MECHANICS

Credits: 5

Course Objectives: To prepare a foundation for advanced study of fluid motion in dimensions of compressible fluid, magneto hydrodynamics and boundary layer theory. Develop concept, models and techniques which enable to solve the problems and help in research in these broad areas.

Course Learning Outcomes: After studying this course the student will be able to

CO1. know about the basics of first and second law of thermodynamics, internal energy, specific heats and concept of entropy, different form of energy equations and dimensional analysis.

CO2. know about compressibility in real fluids, the elements of wave motion, sound wave, shock wave, their formation, properties and elementary analysis.

CO3. understand the interaction between hydrodynamic process and electromagnetic phenomena in term of Maxwell electromagnetic field equation.

CO4. formulate the basic equations of motion in inviscid and viscous conducting fluid flow and be familiar with the Alfven's wave and magneto-hydrodynamic wave.

CO5. know the concepts of boundary layer, boundary layer equations and their solutions with different concept and measurement of boundary layer thickness.

Contents:

UNIT I: Elementary notions of fluid motion: Body forces and surface forces, Nature of stresses, Transformation of stress components, Stress invariants, Principal stresses, Nature of strains, Rates of strain components, Relation between stress and rate of strain components, General displacement of a fluid element, Newton's law of viscosity, Navier-Stokes equation (sketch of proof).

UNIT II: Equation of motion for inviscid fluid, Energy equation, Vortex motion-Helmholtz's vorticity theorem and vorticity equation, Kelvin's circulation Theorem, Mean Potential over a spherical surface, Kelvin's Minimum kinetic energy Theorem, A cyclic irrotational motion.

UNIT III: Wave motion in a gas, Speed of Sound, Equation of motion of a gas, Subsonic, Sonic and Supersonic flows of a gas. Isentropic gas flows.

UNIT IV: Normal and oblique shocks, Plane Poiseuille and Couette flows between two parallel plates, Unsteady flow over a flat plate, Reynold's number.

Mappings: Unit-I implies CO1, Unit-II implies CO2, Unit-III implies CO3, Unit-IV implies CO4.

Books Recommended:

1. L. D. Landau and E. M. Lifshitz, Fluid Mechanics, Butterworth-Heinemann, 2nd Edition, 1987.
2. N. Curle and H. J. Davies, Modern Fluid Dynamics, Vol. I, D. Van Nost. Comp London, 1968.
3. S. W. Yuan, Foundations of Fluid Mechanics, Prentice-Hall, Englewood Cliffs, NJ, 1967.
4. A. S. Ramsey, A Treatise on Hydrodynamics, Part I, G. Bell and Sons Ltd. 1960.
5. F. Chalton, A text book of fluid dynamics. CBS Publication, New Delhi.

Course Objectives: In this course a new algebraic structure, namely, modules is introduced and studied in detail. Modules are the generalization of vector spaces when the underlying field is replaced by an arbitrary ring. The study of modules over a ring also provides an insight into the structure of ring.

Course Learning Outcomes: After studying this course the student will be able to

CO1. identify and construct example of modules, and apply homomorphism theorems on the same.

CO2. distinguish between projective, injective, free, and semi simple modules.

CO3. to understand the chain conditions on modules.

CO4. to characterize finitely generated modules over PID.

Contents:

Unit I: Modules over a ring, Endomorphism ring of an abelian group, R-Module structure on an abelian group M as a ring homomorphism from R to $\text{End}_Z(M)$, submodules, Direct summands, Annihilators, Faithful modules, Homomorphism, Factor modules, Correspondence theorem, Isomorphism theorems, $\text{Hom}_R(M, N)$ as an abelian group and $\text{Hom}_R(M, M)$ as a ring, Exact sequences, Five lemma, Products, coproducts and their universal property, External and internal direct sums.

Unit II: Free modules, Homomorphism extension property, Equivalent characterization as a direct sum of copies of the underlying ring, Split exact sequences and their characterizations, Left exactness of Hom sequences and counter-examples for non-right exactness, Projective modules, Injective modules, Baer's characterization, Divisible groups, Examples of injective modules, Existence of enough injectives.

Unit III: Noetherian modules and rings, Equivalent characterizations, Submodules and factors of noetherian modules, Hilbert basis theorem.

Unit IV: Tensor product of modules, Modules over PID's, semi-simple modules.

Mappings: Unit-I implies CO1, Unit-II implies CO2, Unit-III implies CO3, Unit-IV implies CO4.

Books Recommended:

1. M.F. Atiyah and I.G. MacDonald, *Introduction to Commutative Algebra*, CRC Press, Taylor & Francis, 2018.
2. P.M. Cohn, *Basic Algebra: Groups, Rings and Fields*, Springer, 2005.
3. D.S. Dummit and R.M. Foote, *Abstract Algebra*, Third Edition, Wiley India Pvt. Ltd., 2011.
4. N. Jacobson, *Basic Algebra*, Volumes I & II, Second Edition, Dover Publications, 2009.
5. F. W. Anderson and K. R. Fuller, *Rings and Categories of Modules*, Springer-Verlag.

6. J. S. Golan, Modules and Structures of Rings, Marcel Dekkar Inc.
7. N. S. Gopalakrishnan, University Algebra, Wiley Eastern, New Delhi, 1986.

ME406:

SPECIAL THEORY OF RELATIVITY

Credits: 5

Course Objectives: The aim of this course is to provide overview of concept of relativity.

Course outcomes: After successful completion of this course student will be able to know the necessity of concept of relativity and further improvement of concept of relativity to explain experimental observation. Indeed they are able

CO1:to know the postulates of relativity, relative character of space and time and Lorentz transformations.

CO2:to know the concept of relativistic kinematics length contraction and time dialation.

CO3:to know the geometric representation of space time, Minkowskian space of special relativity.

CO4:to know the variation of mass with velocity and equivalence of mass and energy and relativistic force.

Contents:

UNIT I: Review of Newtonian Mechanics, Inertial frame, Speed of light and Galilean relativity, Michelson- Morley experiment, Lorentz-Fitzerold contraction hypothesis, relative character of space and time, postulates of special theory of relativity, Lorentz transformation equations and geometrical interpretation, Group properties of Lorentz transformations.

UNIT II: Relativistic kinematics, composition of parallel velocities, length contraction, time dilation, transformation equations, equations for components of velocity and acceleration of a particle and contraction factor.

UNIT III: Geometrical representation of space time, four dimensional Minkowskian space of special relativity, time-like intervals, light-like and space- like intervals, Null cone, proper time, world line of a particle, four vectors and tensors in Minkowskian space time.

UNIT IV: Relativistic mechanics- Variations of mass with velocity, equivalence of mass energy, transformation equation for mass, momentum and energy, Energy momentum for light vector, relativistic force and transformation equation for its components, relativistic Lagrangian and Hamiltonian, relativistic equations of motion of a particle, energy momentum tensor of a continuous material distribution.

Mappings: Unit-I implies CO1, Unit-II implies CO2, Unit-III implies CO3, Unit-IV implies CO4.

Books Recommended:

1. C. Mollar, Theory of relativity, Clarendon press, 1952.
2. R. Resnick, Introduction to special relativity, Wiley Eastern Pvt. Ltd. 1972.

3. Satya Prakash, Theory of Relativity.

ME407: REPRESENTATION THEORY OF FINITE GROUPS Credits: 5

Course objectives: Representation theory is the study realization of groups as matrices. It has origin in Algebra and Number Theory. It is vast subject linked to Physics, Harmonic analysis, Combinatorics, Chemistry, Differential Equations, Discrete Mathematics etc.

Course Outcomes: After study of this course, student is able to

CO1: know the representation of groups as matrices/Linear operators.

CO2: know equivalent representations, regular representations and representation of abelian groups.

CO3: understand dimension theorem, Burnside group and Gelfand pairs.

CO4: know induced representations and Tableaux

Unit I : Basic Definitions and Notation, Complex Inner Product Spaces, Further Notions from Linear Algebra , Basic Definitions and First Examples, Maschke's Theorem and Complete Reducibility

Unit II. Morphisms of Representations, The Orthogonality Relations, Characters and Class Functions , The Regular Representation, Representations of Abelian Groups, An Application to Graph theory.

Unit III. A Little Number Theory, The Dimension Theorem, Burnside's, Group, Permutation, The Centralizer Algebra and Gelfand Pairs.

Unit IV. Induced Characters and Frobenius, Induced Representations, Mackey's Irreducibility Criterion, Conjugate Representations, Partitions and Tableaux, Constructing the Irreducible Representations.

Mappings: Unit-I implies CO1, Unit-II implies CO2, Unit-III implies CO3, Unit-IV implies CO4.

Books Recommended:

1. B. Stienberg, Representation Theory of Finite Groups, An Introductory Approach, Springer, 2012.
2. L. Dornhoff, Group Representation Theory, Part A, Marcel Dekker, Inc., New York, 1971.