



**By:**

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**Paper: BED 202- Assessment of Learning**

**Unit: 4**

**Topic: Measures of Central Tendency**

# Measures of Central Tendency (MCT)

## Meaning

MCT is a descriptive statistic that describes the **single average value** (central value or cluster value) of the dataset.

## Characteristics

- i) Based on all scores of data.
- ii) Easy to understand
- iii) Easy to calculate
- iv) Not affected by extreme scores
- v) Sample stability (*different sample gives approx. same measure*)
- vi) Rigidly defined (*clear guideline for calculation*)
- vii) Base of statistical analysis (*without this further statistical calculation is not possible*)

# Learning Objectives

**After studying this unit, the learner will be able to:**

- **Define** the concept of *Measures of Central Tendency* and explain their importance in educational measurement and evaluation.
- **Differentiate** among Mean, Median and Mode on the basis of meaning, calculation and educational use.
- **Select the most appropriate measure** (Mean, Median, or Mode) for a given set of educational data.
- **Interpret educational data** using Mean, Median, and Mode to draw meaningful conclusions about students' performance.
- **Explain the effect of extreme scores** (outliers) on Mean, Median, and Mode with suitable classroom examples.
- **Develop analytical thinking** by comparing results obtained from Mean, Median, and Mode in the same data set.

# Measures of Central Tendency

## Characteristics of a Good MCT

i) **Sensitivity** -: means score quickly affected when make change in value of scores in dataset.

ii) **Stability** -: means score not affected when scores no. are increased or decreased in dataset.

**Note** -: *'Mean' is the most sensitive and most stable score of the MCT.*

## Main objective

1. To condense the **data in single value** means assess the representative value of the all scores.
2. To make comparison of data.
3. Provide help in further statistical analysis.

# Measures of Central Tendency

**Type's of MCT :** *have several type's but mainly 6 type's is*

- i) Mode
  - ii) Median
- } Positional Average
- iii) Arithmetic Mean - **AM**
  - iv) Geometric Mean - **GM** ( *$n^{\text{th}}$  root of the product of the data values*)
  - v) Harmonic Mean - **HM** (*reciprocal of the average of reciprocal*)
  - vi) Quadratic Mean - **QM** (*root mean square*)
- } Mathematical Average

**Note :** *GM, HM, QM mostly used in pure mathematics not in statistics.*

# Measures of Central Tendency

## 1. Mode

**Mode** or **Modal** is a most popular or most frequent score of the dataset and denoted by  $M_o$ .

### Type's

- i) **Unimodal** –: When data consists only single mode.
- ii) **Bimodal** –: When data consists two modes.
- iii) **Multimodal** –: When data consists more than two modes.

### When we use mode

- i) When '**typical** or **tentative**' value is desired.
- ii) When data is lie on **nominal scale**.

# Measures of Central Tendency

## Merits and Demerits of Mode

S.No.	Merits	Demerits
1	Simple and most popular	Uncertain and vague
2	Not affected by extreme scores	Not uses all scores
3	Applicable for both quantitative and qualitative data	Not used for further statistical calculation
4	Graphical presentation	May be not in central position
5	No need of knowing all scores or frequency	More than one in a dataset

# Measures of Central Tendency

## Calculation of Mode

### i) For Ungrouped data

By observation of given scores and select highest frequent score as mode.

**Mode = Observation with maximum frequency**

### ii) For Grouped data :- *By using following formula*

$$M_o = L + \frac{f_a}{f_a + f_b} \times i$$

**Where :**

*L – lower limit of highest frequency class*

*f<sub>a</sub> – frequency of above modal class*

*f<sub>b</sub> – frequency of below modal class*

*i – size of class interval*

# Measures of Central Tendency

## Examples

1. Find the mode of given dataset.

4, 5, 7, 6, 8, 7, 9, 3, 4, 7, 10, 7

### Solution:

In given data 7 is highest appearance number (4 times), So mode is 7.

2. Find the mode of following given data.

Marks Obtained	10-20	20-30	30-40	40-50	50-60
Number of students	5	12	8	5	3

### Solution :

*from table* - maximum frequency class (modal class) is 20-30

- Lower limit of the modal class (L) = 19.5

# Measures of Central Tendency

- Size of the class interval ( $i$ ) = 10
- Frequency of above modal class ( $f_a$ ) = 8
- Frequency of below modal class ( $f_b$ ) = 5
- *Putting these values in the formula-*

$$M_o = L + \frac{f_a}{f_a + f_b} \times i$$

$$M_o = 19.5 + \frac{8}{8+5} \times 10$$

$$M_o = 19.5 + 6.15$$

$$M_o = 25.65$$

- So mode of given data is 25.65.

# Self Assessment Questions

1. Find the mode of following data.

6, 8, 9, 3, 4, 6, 7, 6, 3

2. Find the mode of given data.

Marks Obtained	0-20	20-40	40-60	60-80	80-100
Number of students	5	10	12	6	3

3. A survey on the heights (in cm) of 50 girls of class X was conducted at a school and the following data was obtained:

Height (in cm)	120-130	130-140	140-150	150-160	160-170	Total
Number of girls	2	8	12	20	8	50

# Measures of Central Tendency

## 2. Median:

- Median is the middle value of the dataset, when data are ranked (arranged in ascending or descending order).
- Median is divided the arrange data into **two equal parts**.
- Median is  $P_{50}$  (percentile fifty) of arrange data.
- Median is  $Q_2$  (second quartile) of arrange data.
- Median is  $D_5$  (fifth decile) of arrange data.
- It's **denoted by  $M_d$** .
- **Francis Galton** used the first, english term '**Median**' in **1881**.

### When we use median:

- To find crude average.
- When data is lie on **Ordinal scale**.

# Measures of Central Tendency

## Merits and Demerits of Median

S.No.	Merits	Demerits
1	Easy to compute and understand.	Need to arrange data
2	Not affected by extreme scores	Not uses all scores
3	useful for both quantitative (skewed) and qualitative data	Not accurate when data is small
4	Graphical presentation	May not be exists in data
5	Well defined an ideal average	Affected by fluctuation of sampling

# Measures of Central Tendency

## Calculation of Median

- First we arrange the given data in ascending or descending order, mostly in ascending (from low to high).

### i) For Ungrouped data

- When N is '**Odd**' number :

$$M_d = \left(\frac{N+1}{2}\right) \text{ th score}$$

- When N is '**Even**' number :

$$M_d = \frac{\frac{N}{2} \text{ th score} + \left(\frac{N}{2}+1\right) \text{ th score}}{2}$$

# Measures of Central Tendency

## Calculation of Median

- When Median exists between **Tied Scores**:

$$M_d = L + \frac{\frac{N}{2} - N_B}{f}$$

**Where:**

*L – lower limit of tied score*

*N – total no. of scores (data size)*

*N<sub>B</sub> – Scores below the tied score*

*f – frequency of tied score*

# Measures of Central Tendency

## ii) For Grouped data:

First we find the *cumulative frequency* and *median class* of data then using following formula-

$$M_d = L + \frac{\frac{N}{2} - cf_B}{f} \times i$$

**Where:**

*L* – lower limit of median class

*N* – total no. of scores (data size)

*cf<sub>B</sub>* – cumulative frequency of below median class

*f* – frequency of median class

*i* – class interval size

**Note** – In which class **N/2 th cumulative frequency** exists, called median class.

# Measures of Central Tendency

## Examples:

1. Find the median of given dataset.

5, 7, 6, 8, 9, 3, 10

## Solution:

Firstly, arrange data in ascending order

3, 5, 6, 7, 8, 9, 10

**So n = 7 (odd no.)**

Then use following formula-

$$M_d = \left(\frac{N+1}{2}\right) \text{ th score}$$

$$M_d = \left(\frac{7+1}{2}\right) \text{ th score}$$

$$M_d = \left(\frac{8}{2}\right) \text{ th score}$$

$$M_d = 4^{\text{th}} \text{ score}$$

**So Median of following data is 7.**

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# Measures of Central Tendency

2. Find the median of following given data.

Marks Obtained	10-20	20-30	30-40	40-50	50-60
Number of students	5	8	12	5	3

**Solution:**

Class Interval	Frequency (f)	Cumulative Frequency (cf)
50-60	3	33
40-50	5	30
<b>30-40</b> (Md class)	12	25
20-30	8	13
10-20	5	5
<b>i = 10</b>	<b>N = 33</b>	<b>N/2 = 16.5</b>

*From following table:*

$$L = 29.5 \quad N/2 = 16.5 \quad cf_b = 13 \quad f = 12 \quad i = 10$$

# Measures of Central Tendency

*Putting these values in the formula-*

$$M_d = L + \frac{\frac{N}{2} - cfB}{f} \times i$$

$$M_d = 29.5 + \frac{16.5 - 13}{12} \times 10$$

$$M_d = 29.5 + \frac{3.5}{12} \times 10$$

$$M_d = 29.5 + \frac{35}{12}$$

$$M_d = 29.5 + 2.92$$

$$M_d = 32.42$$

**So median of given dataset is 32.42**

## Self Assessment Questions

1. Find the median of given dataset.

60, 49, 50, 45, 77, 72, 80, 89, 65      (Ans. 65)

2. Find the median of given dataset.

65, 77, 82, 72, 90, 42, 64, 89      (Ans. 74.5)

3. Find the median of given data.

C.I	10-19	20-29	30-39	40-49	50-59	60-69	70-79	80-89	90-99
f	1	2	7	12	23	25	18	10	2

Ans. 61.50

# Measures of Central Tendency

## 3. Mean

- Mean is the '**center of gravity**' of dataset.
- It's also called '**Arithmetic Average**' because obtained by adding up all the scores and dividing by the total number of scores.
- Denoted by M.

### Characteristics:

- The sum of each score's distance from the mean is zero.
- Express the **size of score**.
- It may not be an actual score in the distribution.
- Applied on **interval scale** or **ratio scale**. (symmetrical and continuous scores )

# Measures of Central Tendency

## Merits and Demerits of Mean

S.No.	Merits	Demerits
1	One and only one mean of any dataset	Not determined by graphical location
2	Most reliable MCT ( <b>based on interval and ratio scale</b> )	Not applicable in qualitative data
3	Least affected by sampling fluctuation ( <b>highest stable</b> )	Seriously affected by extreme scores
4	Quickly affected make change in value of scores ( <b>highest sensitive</b> )	May not be exists in data
5	Based on all scores of data	If any one score is missing, mean can't be calculated

# Measures of Central Tendency

## When we use Mean

- Sampling stability is desired
- Required further statistical value such as standard deviation, coefficient of variation and skewness, correlation etc.

## Calculation of Mean

### i) For Ungrouped data (small data)

Adding all the scores and dividing by the total number of scores.

$$M = \frac{\sum X}{N}$$

**Where:**

$\sum X$  – Sum of all scores

$N$  – Total No. of scores

# Measures of Central Tendency

ii) For Grouped data (large data)

a) *Direct method*

$$M = \frac{\sum fX}{N}$$

**Where:**

*f* – frequency of class

*X* – mid point of class

$\sum fX$  – sum of product of all class mid point and class frequency

*N* – total no. of frequency or size of data

# Measures of Central Tendency

## b) Indirect method

$$M = A.M. + \frac{\sum fd}{N} \times i$$

**Where:**

*A.M. – assumed mean (mid point of middle class)*

*f – class frequency*

*d – class deviation*

*$\sum fd$  – sum of product of all class deviation and frequency*

*i – class interval*

*N- size of group or total of frequency*

**Note :** Calculation of d is based on following formula :  $d = \frac{X - A.M.}{i}$

# Measures of Central Tendency

- **Combined Mean** (mean from two or more groups)

$$M_{\text{comb.}} = \frac{N_1 M_1 + N_2 M_2 + \dots + N_i M_i}{N_1 + N_2 + \dots + N_i}$$

Where:

$M_1, M_2, \dots, M_i$  – Mean of various groups

$N_1, N_2, \dots, N_i$  – Size of various groups

- **Empirical relation between Mean, Median and Mode**

$$\text{Mode} = 3\text{Median} - 2\text{Mean}$$

$$M_o = 3M_d - 2M$$

## Note :

1. If we **add or subtract** any value in data then mean sequentially **increased or decreased**
2. If we **multiply** any value in given data then mean **increased  $n^{\text{th}}$  times**.

# Measures of Central Tendency

## Applicability of MCTs on scales

### 1. Nominal scale:

- Only mode can be used.

### 2. Ordinal scale:

- Mode and the median may be used.
- The median provides more information in comparison to mode (taking into account the ranking of categories.)

### 3. Interval and Ratio scale:

- Mode, median, and mean may all be calculated.
- The mean provides the most information about the distribution, but the median is preferred if the distribution is skewed.

# Measures of Central Tendency

## Examples :

1. Find the mean of following data.

12, 32, 15, 19, 28, 42, 40

## Solution:

$$\sum X = 12+32+15+19+28+42+40 = 188$$

$$N = 7$$

*Putting these values in following formula*

$$M = \frac{\sum X}{N}$$

$$M = \frac{188}{7}$$

$$M = 26.86$$

**Mean of given data is 26.86**

2. Find the mean of given dataset

CI	0-4	5-9	10-14	15-19	20-24	25-29
f	3	7	12	5	4	2

# Measures of Central Tendency

**Solution:**

C.I	f	X	fX
25-29	2	27	54
20-24	4	22	88
15-19	5	17	85
10-14	12	12	144
5-9	7	7	49
0-4	3	2	6
	N = 33		$\Sigma fX = 426$

*Putting following values in formula*

$$M = \frac{\Sigma fX}{N}$$

$$M = \frac{\Sigma 426}{33}$$

$$M = 12.91$$

**So mean of given data is 12.91**

# Measures of Central Tendency

3. Find the mean of given dataset.

C.I	0-4	5-9	10-14	15-19	20-24	25-29	30-34
f	3	5	7	10	5	4	1

Solution:

C.I.	f	d	fd
30-34	1	+3	+3
25-29	4	+2	+8
20-24	5	+1	+5 (+16)
<b>15-19 (A.M.)</b>	10	0	0
10-14	7	-1	-7
5-9	5	-2	-10
0-4	3	-3	-9 (-26)
<b>i = 5</b>	<b>N = 35</b>		<b><math>\sum fd = -10</math></b>

A.M. = 17

# Measures of Central Tendency

*Putting following values in formula:*

$$M = A.M. + \frac{\sum fd}{N} \times i$$

$$M = 17 + \frac{-10}{35} \times 5$$

$$M = 17 + \frac{-50}{35}$$

$$M = 17 + (-1.43)$$

$$M = 15.57$$

**So mean of given data is 15.57**

# Self Assessment Questions

1. Find mean of following dataset.

31, 34, 28, 36, 33, 34, 26, 29, 26, 39, 37, 37, 32, 35, 39 (Ans. 33.07)

2. Find the mean of given table.

C.I.	10-14	15-19	20-24	25-29	30-34	35-39	40-44
f	5	7	10	15	5	4	1

3. Find the mean, median and mode of given dataset.

C.I.	10-20	20-30	30-40	40-50	50-60	60-70	70-80
f	3	5	8	10	7	4	2

4. If three individual groups mean and size are sequentially 15.4, 16.5, 18.5 and 60, 40, 50 then find combined mean of three groups .

5. If a distribution mean and mode values sequentially 22.5 and 15, find the median value of that distribution.

**Thank You for  
Reading**

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