M.Sc. 3sem Optoelectronics unit 2 (part 1)

The Manley–Rowe Relations

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The Manley–Rowe Relations

Let us now consider, the mutual interaction of three optical waves propagating through a lossless nonlinear optical medium as shown in Fig.-1

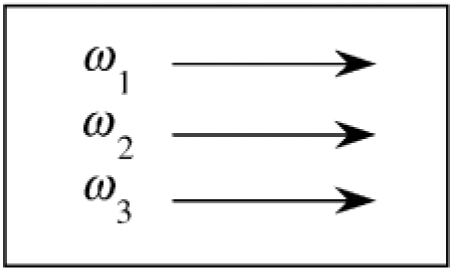

Fig.-1 Optical waves of frequencies ω_1, ω_2, and $\omega_3=\omega_1+\omega_2$ interact in a lossless nonlinear optical medium

Let us now consider the spatial variation of the intensity associated with each of these waves

$$I_i = 2n_i\epsilon_0 c A_i A_i^* \tag{1}$$

The variation of the intensity is described by

$$\frac{dI_i}{dz} = 2n_i\epsilon_0 c\left(A_i^*\frac{dA_i}{dz} + A_i\frac{dA_i^*}{dz}\right) \tag{2}$$

By using above result and

$$\frac{dA_1}{dz} = \frac{2id_{\text{eff}}\omega_1^2}{k_1c^2}A_3A_2^*e^{-i\Delta kz} \tag{3}$$

We find that the spatial variation of the intensity of the wave at frequency $\boldsymbol{\omega_1}$,

$$\frac{dI_1}{dz} = 2n_i\epsilon_0 c\frac{2d_{\text{eff}}\omega_1^2}{k_1c^2}\left(iA_1^*A_3A_2^*e^{-i\Delta kz} + \text{c.c.}\right)$$
$$= 4\epsilon_0 d_{\text{eff}}\omega_1\left(iA_3A_1^*A_2^*e^{-i\Delta kz} + \text{c.c.}\right) \tag{4}$$

$$\frac{dI_1}{dz} = -8\epsilon_0 d_{\text{eff}}\omega_1 \operatorname{Im}\left(A_3A_1^*A_2^*e^{-i\Delta kz}\right) \tag{5}$$

Similarly, the spatial variation of the intensities of the waves at frequencies $\boldsymbol{\omega_2}$ and $\boldsymbol{\omega_3}$ is given by

$$\frac{dI_2}{dz} = -8\epsilon_0 d_{\text{eff}}\omega_2 \operatorname{Im}\left(A_3A_1^*A_2^*e^{-i\Delta kz}\right) \tag{6}$$

$$\frac{dI_3}{dz} = -8\epsilon_0 d_{\text{eff}}\omega_3 \operatorname{Im}\left(A_3^*A_1A_2e^{i\Delta kz}\right)$$
$$= 8\epsilon_0 d_{\text{eff}}\omega_3 \operatorname{Im}\left(A_3A_1^*A_2^*e^{-i\Delta kz}\right). \tag{7}$$

From the above equations it can be seen that the sign of $\mathbf{dI_1/dz}$ is same as that of $\mathbf{dI_2/dz}$ but is opposite to that of $\mathbf{dI_3/dz}$. It is also seen that the direction of energy flow depends on the relative phases of the three interacting fields.

The set of Eqs. (5), (6) and (7) shows that the total power flow is conserved, as expected for propagation through a lossless medium. To demonstrate this fact,the total intensity is defined by

$$\frac{dI}{dz} = \frac{dI_1}{dz} + \frac{dI_2}{dz} + \frac{dI_3}{dz}$$
$$= -8\epsilon_0 d_{\text{eff}}(\omega_1 + \omega_2 - \omega_3) \operatorname{Im}\left(A_3A_1^*A_2^*e^{i\Delta kz}\right) = 0 \tag{8}$$

We have made use of set of Eqs. (5), (6) and (7) and where the last equality follows from the fact that $\omega_3=\omega_1+\omega_2$.

The Eqs. (5), (6) and (7) also implies that

$$\frac{d}{dz}\left(\frac{I_1}{\omega_1}\right)=\frac{d}{dz}\left(\frac{I_2}{\omega_2}\right)=-\frac{d}{dz}\left(\frac{I_3}{\omega_3}\right) \tag{9}$$

These equalities are known as the Manley–Rowe relations (Manley and Rowe, 1959). Since the energy of a photon of frequency ω_i is $\hbar\omega_i$, the quantity I_i/ω_i appears in these relations is proportional to the intensity of the wave measured in photons per unit area per unit time. The Manley–Rowe relations can be expressed as

$$\frac{d}{dz}\left(\frac{I_2}{\omega_2}+\frac{I_3}{\omega_3}\right)=0,\qquad \frac{d}{dz}\left(\frac{I_1}{\omega_1}+\frac{I_3}{\omega_3}\right)=0,\qquad \frac{d}{dz}\left(\frac{I_1}{\omega_1}-\frac{I_2}{\omega_2}\right)=0. \tag{10}$$

Integrating these Eqs. to obtain the three conserved quantities M_1, M_2, and M_3, which are given by

$$M_1=\frac{I_2}{\omega_2}+\frac{I_3}{\omega_3},\qquad M_2=\frac{I_1}{\omega_1}+\frac{I_3}{\omega_3},\qquad M_3=\frac{I_1}{\omega_1}-\frac{I_2}{\omega_2} \tag{11}$$

From Eq.(11), we find that the rate at which photons at frequency ω_1 are created is equal to the rate at which photons at frequency ω_2 are created and is equal to the rate at which photons at frequency ω_3 are destroyed. This result can be understood by means of the energy level description of a three-wave mixing process as shown in Fig.-2.

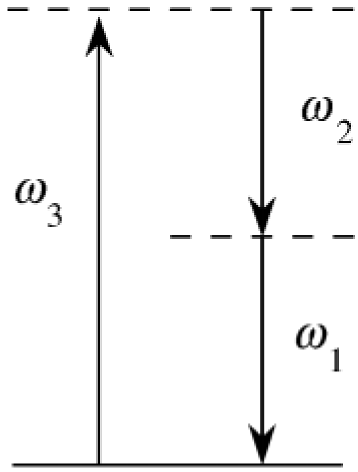

Fig.-2 Photon description of the interaction of three optical waves

This diagram shows that, for a lossless medium, the creation of $\boldsymbol{\omega_1}$ photon must be accompanied by the creation of $\boldsymbol{\omega_2}$ photon and the annihilation of $\boldsymbol{\omega_3}$ photon. It is seen that the Manley–Rowe relations should be consistent with this quantum-mechanical interpretation, where derivation of these relations appears to be entirely classical. It should be noted that this derivation assumes that the nonlinear susceptibility possesses permutation symmetry in which the coupling constant $\mathbf{d_{eff}}$ having the same value in each of the coupled-amplitude equations

$$\frac{dA_1}{dz} = \frac{2i d_{\text{eff}} \omega_1^2}{k_1 c^2} A_3 A_2^* e^{-i\Delta k z} \quad \textbf{(12)}$$

$$\frac{dA_2}{dz} = \frac{2i d_{\text{eff}} \omega_2^2}{k_2 c^2} A_3 A_1^* e^{-i\Delta k z} \quad \textbf{(13)}$$

$$\frac{dA_3}{dz} = \frac{i\omega_3}{2\epsilon_0 n_3 c} p_3 e^{i\Delta k z} \quad \textbf{(14)}$$

Where $P_3 = p_3 \exp[i(k_1 + k_2)z]$ is slowly varying amplitude of the nonlinear polarization.