Lagrange polynomial

(Lagrange's Interpolation formula for unequal Intervals)

In numerical analysis, **Lagrange polynomials** are used for polynomial interpolation. For a given set of points with no two values equal, the Lagrange polynomial is the polynomial of lowest degree that assumes at each value the corresponding value , so that the functions coincide at each point.

Table Given:

x	x0	x1	x2	x3
f(x)	f(x0)	f(x1)	f(x2)	f(x3)

Polynomial=?

P(x) =L0(x).f(x0)+L1(x).f(x1)+L2(x).f(x2)+L3(x).f(x3)...............eq.

L0(x) = (x-x1)(x-x2)(x-x3) / (x0-x1)(x0-x2)(x0-x3)

L1(x) = (x-x0)(x-x2)(x-x3) / (x1-x0)(x1-x2)(x1-x3)

L2(x) = (x-x0)(x-x1)(x-x3) / (x2-x0)(x2-x1)(x-x3)

L3(x) =(x-x0)(x-x1)(x-x2) / (x3-x0)(x3-x1)(x3-x2)

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x	x0	x1	x2
f(x)	f(x0)	f(x1)	f(x2)

P(x)=L0(x).f(x0)+L1(x).f(x1)+L2(x).f(x2).............eq.

L0(x)=(x-x1)(x-x2)/(x0-x1)(x0-x2)

L1(x)=(x-x0)(x-x2)/(x1-x0)(x1-x2)

L2(x)=(x-x0)(x-x1)/(x2-x0)(x2-x1)

e.g.

Find Lagrange's interpolating polynomial from following table:

x	1	2	3
f(x)	2	3	4

$X0=1$

$X1=2$

$X2=3$

$f0=f(x0) =2$

$f1=f(x1) =3$

$f2=f(x2) =4$

L0(x) =(x-x1)(x-x2)/(x0-x1)(x0-x2) = (x-2)(x-3)/(1-2)(1-3) = (x^2 -5x + 6)/((-1)(-2))=(x^2 -5x + 6)/2

L1(x) =(x-x0)(x-x2)/(x1-x0)(x1-x2) = (x-1)(x-3)/(2-1)(2-3) = (x^2 -4x + 3)/ (-1) = -(x^2 -4x + 3)

L2(x) =(x-x0)(x-x1)/(x2-x0)(x2-x1) = (x-1)(x-2)/(3-1)(3-2) = (x^2 -3x + 2) /2

P(x) =L0(x)f0 + L1(x)f1 +L2(x)f2

= ((x^2 -5x + 6)/2) * 2 -(x^2 -4x + 3) * 3 +((x^2 -3x + 2) /2) *4

=x^2 -5x + 6-3x^2+12x-9+2x^2 -6x + 4

P(x) =x+1

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Q. Find Lagrange's interpolating polynomial for following data:

x	3	2	1
f(x)	2	3	4

Also find the value of function at x=7 and x=10.

L0(x)=(x-x1)(x-x2) / (x0-x1)(x0-x2) =(x-2)(x-1) /(3-2)(3-1) = (x^2-3x+2) /2

L1(x)=(x-x0)(x-x2) / (x1-x0)(x1-x2) =(x-3)(x-1) /(2-3)(2-1) = (x^2-4x+3) /-1

L2(x)=(x-x0)(x-x1) / (x2-x0)(x2-x1) =(x-3)(x-2) /(1-3)(1-2) = (x^2-5x+6) /2

F(x)=L0(x)*f0 + L1(x)*f1 +L2(x)*f2

=(x^2-3x+2) /2 * 2 + (x^2-4x+3) /-1 * 3 + (x^2-5x+6) /2 *4

=x^2-3x+2-3x^2+12x-9+2x^2-10x+12

= -x+5

f(x)=5-x Ans.

f(7)=5-7=-2 Ans.

f(10)=5-10=-5 Ans.

Q. Find the value at x=12 from following data using Lagrange's are interpolating polynomial

x	-3	-2	-1
f(x)	2	3	4

Method 1:

Polynomial find then x=12 put

Method 2:

Direct x=12 put

===============================Method 1:===

x	-3	-2	-1
f(x)	2	3	4

$L0(x)=(x^2 +3x +2)/2$

$L1(x)=(-x^2 -4x -3)$

$L2(x)=(x^2+5x+6)/2$

P(x)=(After solving………………)=x+5

P(12)=12+5=17 Ans.

===============================Method 2:===

x	-3	-2	-1
f(x)	2	3	4

L0(12)=91

L1(12)=-195

L2(12)=105

P(12)=91*2+(-195)*3+105*4 = 17 Ans.
