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Unit: 4

Topic: Graphical Presentation of Data

Learning Objectives

(अधिगम उद्देश्य)

After studying this unit, the learner will be able:

- **To understand** the meaning and importance of graphical presentation of data.
- **To explain** the historical development of graphical presentation from the Cartesian coordinate system to modern graphical methods.
- **To develop** understanding of the coordinate system, including X-axis, Y-axis, origin, quadrants, and scale.
- **To understand** the proper use of scales and broken-axis symbols in graphs.
- **To understand** different types of graphs such as bar graphs, line graphs, pie charts, histograms, frequency polygons, frequency curves, and ogives.

Learning Objectives

(अधिगम उद्देश्य)

After studying this unit, the learner will be able:

- **To explain the difference** between various graphical methods used in statistics.
- **To learn the process** of constructing graphs using suitable scales and data.
- **To understand** positively skewed, negatively skewed, symmetrical, bimodal, uniform, and random distributions.
- **To develop** the ability to interpret and analyze statistical data through graphs.
- **To apply** graphical presentation techniques in research, education, and daily life situations.

Graphical Presentation of Data: GPD

(आंकड़ों का आलेखीय निरूपण)

- **Meaning:** Graphical presentation of data means representing data by using graphs, charts and diagrams to make it easy to understand.
- **In other words,** GPD is the method of representing numerical information visually using graphs, charts or diagrams, so that the data can be understood easily, compared quickly and interpreted effectively.

➢ वह प्रक्रिया है जिसमें आंकड़ों को चित्र, ग्राफ या आरेख के माध्यम से प्रस्तुत किया जाता है ताकि उन्हें आसानी से समझा जा सके.

Objectives of GPD

(आलेखीय निरूपण के उद्देश्य)

I. To simplify complex data

(जटिल आंकड़ों को सरल बनाना)

II. To make easy comparison of data

(आंकड़ों की तुलना को आसान बनाना)

III. To show trends and patterns of data

(आंकड़ों की प्रवृत्तियों को प्रदर्शित करना)

IV. To save time and effort in calculation

(गणना में समय एवं श्रम की बचत करना)

Uses & Importance of GPD

(आलेखीय निरूपण के उपयोग एवं महत्व)

I. Helps in Quick Decision Making

(शीघ्र निर्णय लेने में सहायक)

II. Attractive and Easy to Interpret

(आकर्षक एवं समझने में आसान)

III. Useful in Research and Statistics

(शोध एवं सांख्यिकी में उपयोगी)

IV. Helps in Prediction

(भविष्यवाणी में सहायक)

Chronological History of Graphical Presentation of Data

(आंकड़ों का आलेखीय निरूपण का कालानुक्रमिक इतिहास)

➢ Early Stage (Before 17th Century)

- Data was presented only in **tables and written form** and no graphical representation

➢ Foundation: Coordinate System (17th Century)

- French Mathematician '**René Descartes**' developed the Cartesian Coordinate System and introduced two perpendicular axes: X-axis (Abscissa)⟶ horizontal line and Y-axis (Ordinate)⟶ vertical line.
- Intersection point is called the Origin (0,0) and plane is divided into four quadrants:

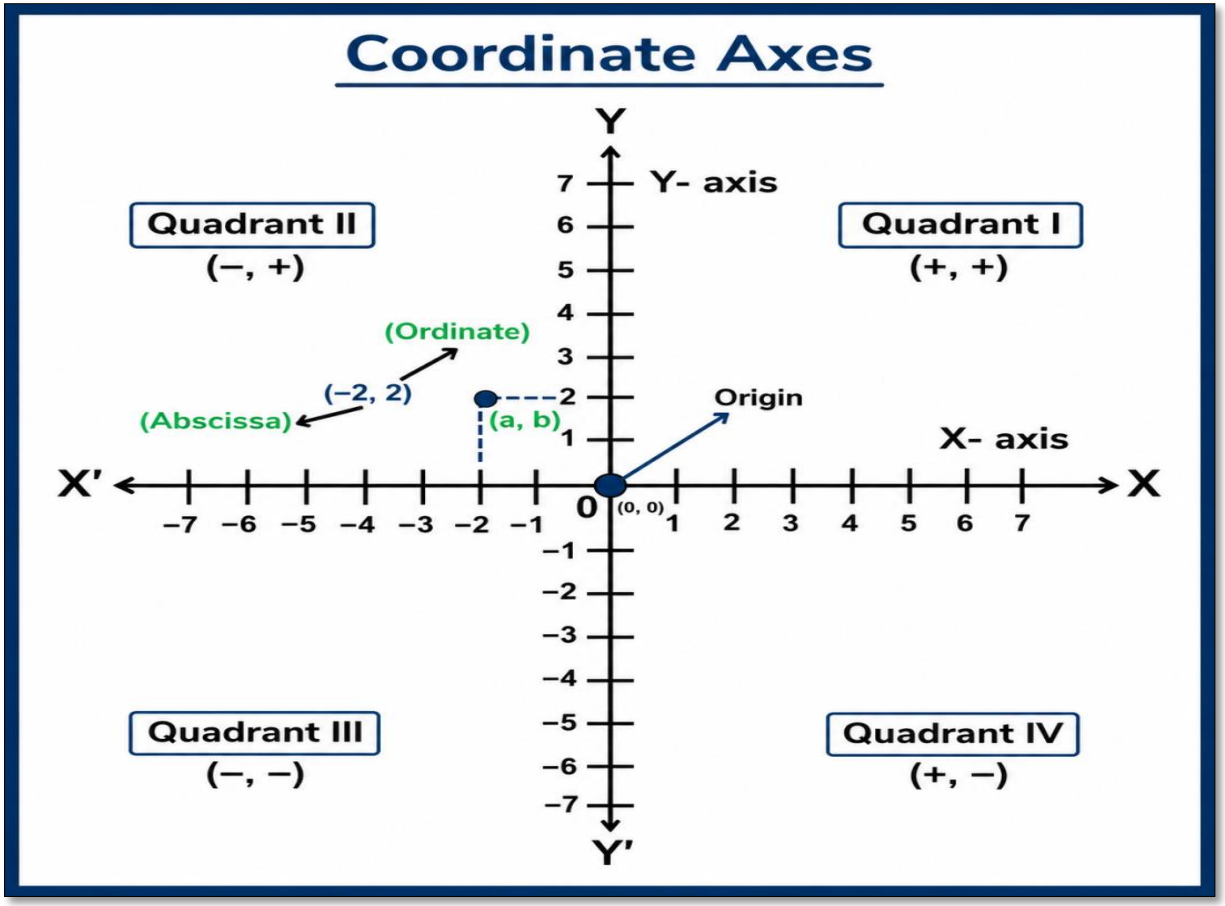
i. (+, +)

ii. (−, +)

iii. (−, −)

iv. (+, −)

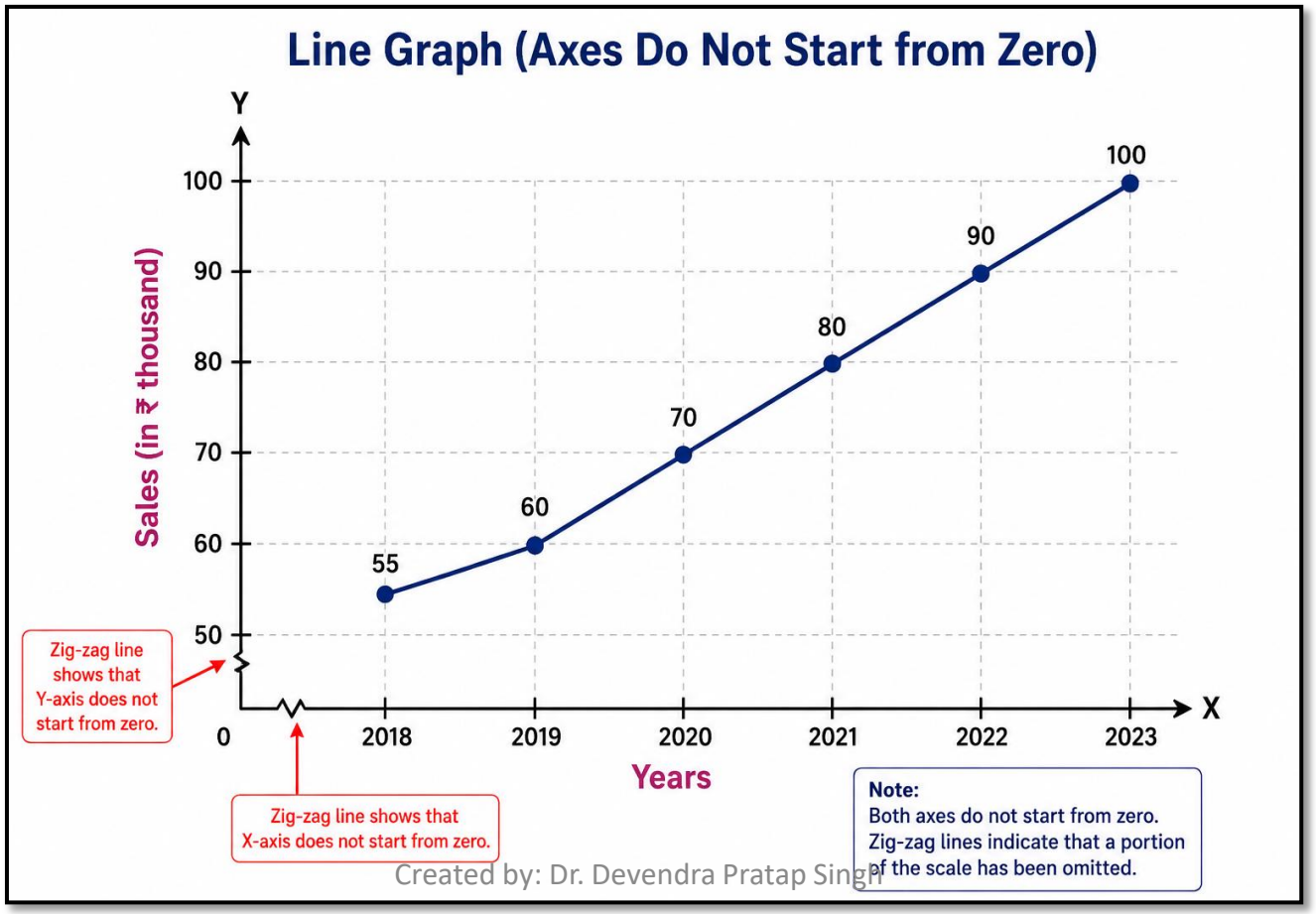
Coordinate System (निर्देशांक अक्ष)



- The length of X-axis and Y-axis should be proportional in (3:2) or (4:3) proportion.

Zig-Zag line in Axes

- When the X-axis or Y-axis does not start from zero, a break mark (zig zag sign // or ~) is made on the axis line to indicate that a portion of the scale has been omitted.



Chronological History of Graphical Presentation of Data

➢ Beginning of Graphical Methods (1786)

- William Playfair introduced-
  - Bar Graph
  - Line Graph
- First book using graphs: **The Commercial and Political Atlas**

➢ Development of Pie Chart (1801)

- Introduced by William Playfair to showed proportions.

➢ Statistical Graphics Expansion (19th Century)

- Florence Nightingale used graphs (coxcomb diagram) for health and social data analysis.

Chronological History of Graphical Presentation of Data

➢ Growth of Statistical Graphs (Early 20th Century)

- Development of Histogram
- Frequency Polygon
- Ogive (Cumulative Curve)

➢ Exploratory Data Analysis (1970s)

- John Tukey introduced-
  - Box Plot
  - Concept of EDA (Exploratory Data Analysis)

➢ Computer Era (Late 20th Century)

- Graphs became Easy to create, Fast and Accurate with Software like Excel and others.

Chronological History of Graphical Presentation of Data

➢ Modern Digital Era (21st Century)

- Use of-
  - Interactive Graphs
  - Dash Boards
  - Data Visualization Tools (Power BI, Tableau)

➢ Summary

Descartes (Axes) ⟶ **William Playfair** (Bar & Line Graph)⟶ **William Playfair** (Pie Chart) ⟶ **Statistical Graphs** (Histogram, Polygon)⟶ **John Tukey** (Box Plot & EDA) ⟶ **Computer Era** ⟶ **Digital Era**

Types of Graphical Presentation
(आलेखीय निरूपण के प्रकार)

1. Bar Graph (स्तंभ आरेख)
2. Line Graph (रेखा आरेख)
3. Pie Chart (वृत्त आरेख)
4. Histogram (आवृत्ति आयतचित्र)
5. Frequency Polygon (आवृत्ति बहुभुज)
6. Frequency Curve (आवृत्ति वक्र)
7. Ogive: Cumulative Frequency Curve (तोरण: संचयी आवृत्ति वक्र)

1. Bar Graph (स्तंभ आरेख)

Definition: Bar graph is a graphical representation of data in which rectangular bars are used to show different values. The height or length of each bar is proportional to the value it represents.

परिभाषा: स्तंभ आरेख एक ऐसा आलेखीय निरूपण है जिसमें आंकड़ों को आयताकार स्तंभों (bars) के रूप में प्रदर्शित किया जाता है। प्रत्येक स्तंभ की ऊँचाई या लंबाई उसके मान (value) को दर्शाती है।

Types of Bar Graph
(स्तंभ आरेख के प्रकार)

I. Simple Bar Graph

(साधारण स्तंभ आरेख)

II. Multiple Bar Graph

(बहु स्तंभ आरेख)

III. Component Bar Graph

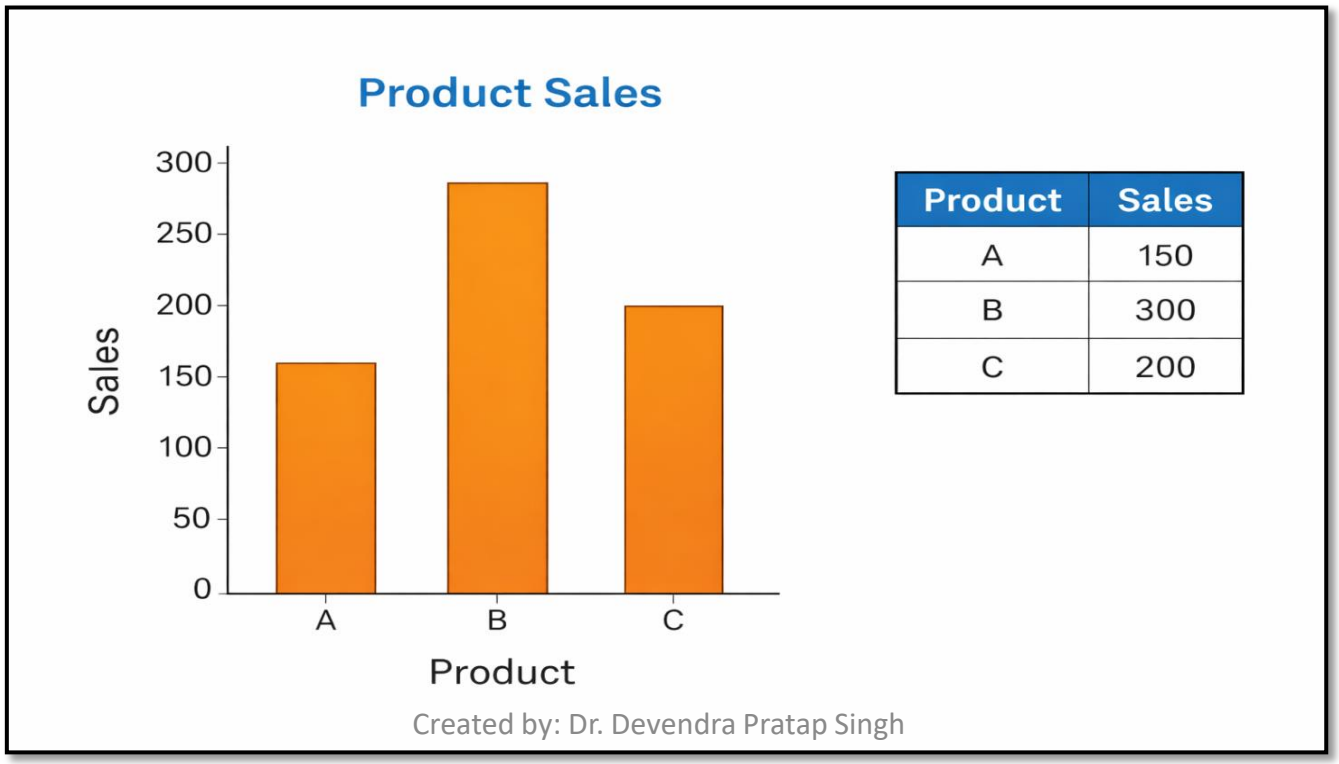
(घटक स्तंभ आरेख)

Simple Bar Graph (साधारण स्तंभ आरेख)

- Simple Bar Graph represents only **one variable** data means each category has only a single bar.
- साधारण आरेख केवल एक ही चर के डेटा को प्रदर्शित करता है अर्थात इसमें प्रत्येक श्रेणी के लिए केवल एक आयताकार बार होता है।

✓ **Use:** For direct comparison of different items.

✓ **Example:** Graph of sales of any three products.



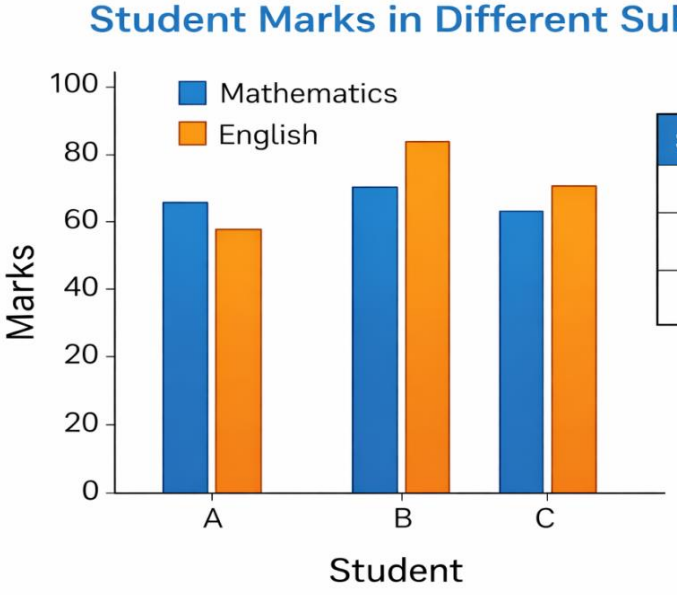
Product	Sales
A	150
B	300
C	200

Multiple Bar Graph (बहु स्तंभ आरेख)

- Multiple bar graph is used to compare **two or more related variables**. In this, bars of different colours are drawn side by side for each category.
- यह आरेख दो या दो से अधिक संबंधित चरों की एक साथ तुलना करने के लिए उपयोग किया जाता है। इसमें प्रत्येक श्रेणी के लिए अलग-अलग रंगों वाले बार साथ-साथ बनाए जाते हैं।

✓ **Use:** To compare between two or more groups.

✓ **Example:** Marks of three students in two subjects.



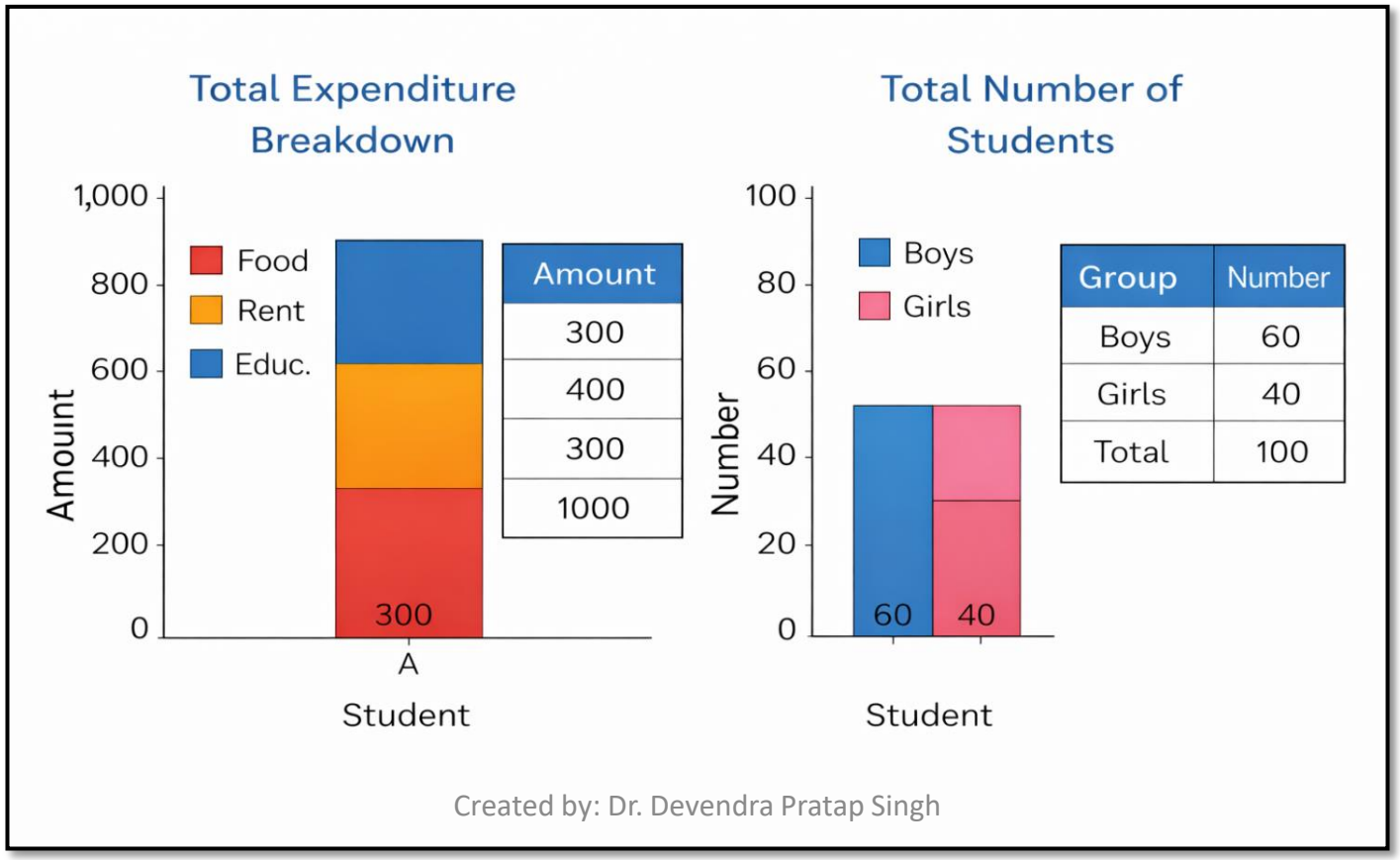
Student	Mathematics	English
A	80	60
B	70	90
C	60	80

Component Bar Graph (घटक स्तंभ आरेख)

- Component bar graph shows the **division of a total into parts** within a single bar.
- घटक स्तंभ आरेख, एक ही स्तंभ में कुल योग को विभिन्न भागों में विभाजित करके प्रदर्शित करता है।

✓ **Use:** To show the total divided into sub categories.

✓ **Example:** Total expenditure divided into food, rent and education.



2. Line Graph (रेखीय आरेख)

- Line graph is a type of graphical representation in which data points are plotted **on a coordinate plane** and **connected by straight lines**.
- लाइन ग्राफ वह आरेख है जिसमें डेटा बिंदुओं को निर्देशांक तल (coordinate plane) पर अंकित किया जाता है एवं सभी को सीधी रेखाओं से जोड़ा जाता है।

➢ Key Points:

- Horizontal line of graph (X-Axis) usually represents time or categories .
- Vertical line of graph (Y-Axis) shows values.

➢ Use:

- Mainly used to show **changes or trends** over the time.

Type of Line Graph
(रेखीय आरेख के प्रकार)

i. Simple Line Graph

(साधारण रेखीय आरेख)

ii. Multiple Line Graph

(बहु रेखीय आरेख)

iii. Compound Line Graph: Area Graph

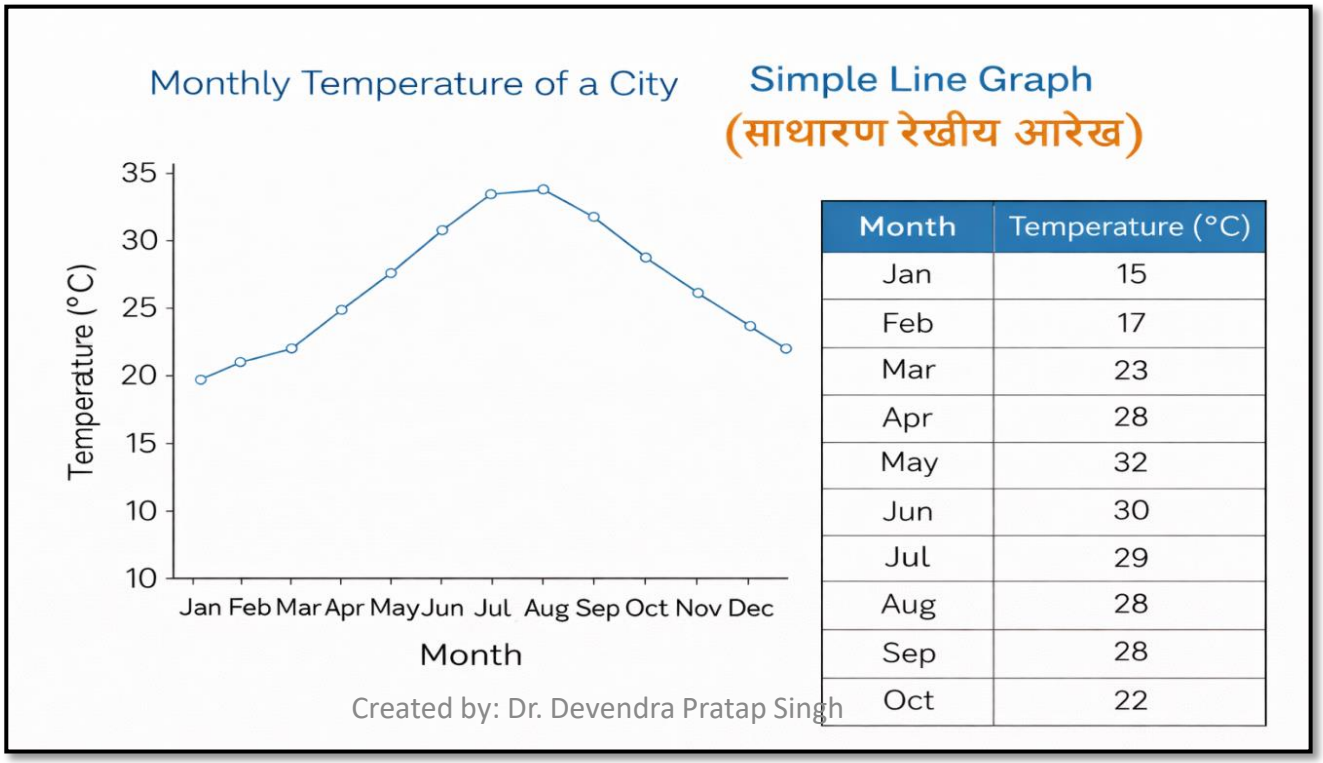
(संयुक्त रेखीय आरेख: क्षेत्र आरेख)

Simple Line Graph (साधारण रेखीय आरेख)

- Simple line graph represents **one set of data**. It shows how a single variable continuous **changes over time** by connecting data points with a line.
- साधारण रेखीय आरेख में केवल एक ही डेटा सेट को दर्शाया जाता है। इसमें बिंदुओं को रेखा से जोड़कर किसी एक चर के समय के साथ परिवर्तन को दिखाया जाता है।

✓ **Use:** Clearly shows the trend.

✓ **Example:** Monthly temperature of a city.



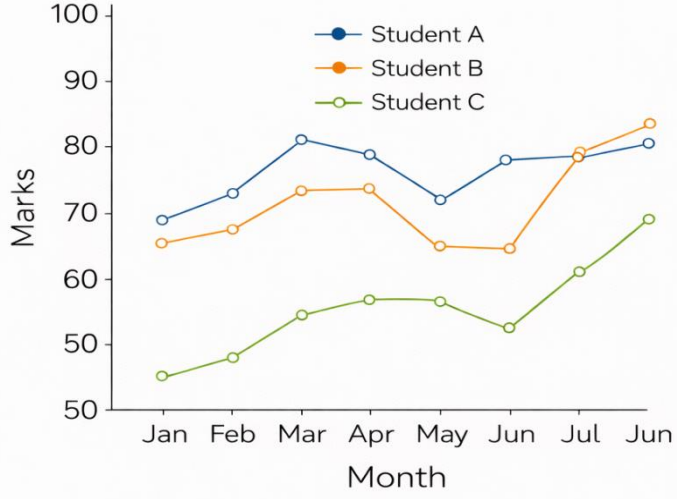
Month	Temperature (°C)
Jan	15
Feb	17
Mar	23
Apr	28
May	32
Jun	30
Jul	29
Aug	28
Sep	28
Oct	22

Multiple Line Graph (बहु रेखीय आरेख)

- Multiple line graph represents **two or more sets of data** on the same graph using different lines.
- बहु रेखीय आरेख में दो या अधिक डेटा सेट को एक ही ग्राफ पर अलग-अलग रेखाओं के माध्यम से दर्शाया जाता है।

✓ **Use:** Helps to compare trends between variables over time with different colours lines.

✓ **Example:** Marks of three students in different months test.



Multiple Line Graph
(बहु रेखा आरेख)

Month	Student A	Student B	Student C
Jan	85	78	65
Feb	88	74	69
Mar	92	82	70
Apr	81	76	80
May	90	84	73
Jun	90	84	73
Jun	86	88	79
Jun	86	88	79

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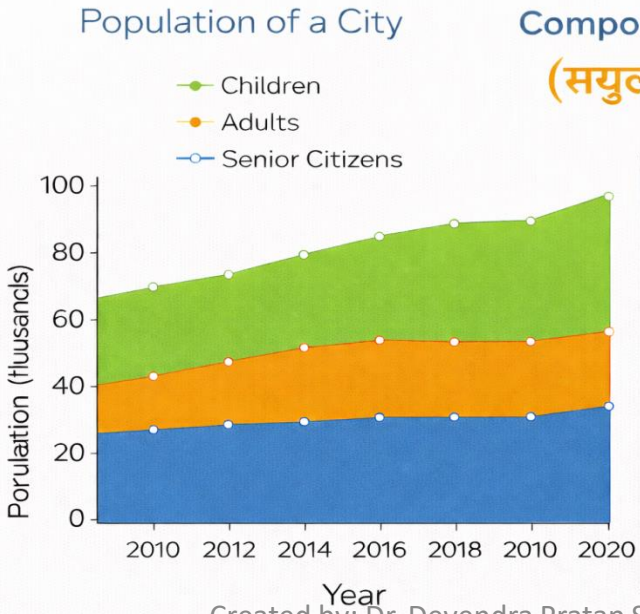
Compound Line Graph: Area Graph (संयुक्त रेखीय आरेख: क्षेत्र आरेख)

- Compound line graph (area graph) is a type of graph in which **multiple data sets are represented by lines and the area between them is shaded** to show the total as well as its components over time.
- वह ग्राफ है जिसमें एक से अधिक डेटा सेट को रेखाओं द्वारा दर्शाया जाता है एवं उनके बीच का क्षेत्र छायांकित किया जाता है, जिससे कुल और उसके भागों को समय के साथ प्रदर्शित किया जा सके।

✓ **Uses:** To show total along with its components and study the changes over time.

✓ **Example:** Population of a city divided into children, adults and senior citizens over different years.



Compound Line Graph: Area Graph
(सयुक्त रेखीव आरेख: क्षेत्र आरेख)

Year	Children	Adults	Senior Citizens
2010	20	40	10
2012	21	43	11
2014	23	46	12
2016	23	48	12
2016	23	48	13
2018	22	50	15
2018	21	52	18
2020	21	52	18

3. Pie Chart (वृत्त आरेख)

- Pie chart is a circular graph that is divided into different sectors (slices), where each sector represents a proportion or percentage of the total data.
- वृत्त आरेख एक गोलाकार आरेख होता है, जिसमें पूरे वृत्त को विभिन्न भागों (खण्डों) में विभाजित किया जाता है। जहाँ प्रत्येक खण्ड कुल आंकड़ों के एक अनुपात या प्रतिशत को दर्शाता है।
- **Key Points:**

✓ Total value = 100%

✓ Each slice represents a part of the whole

✓ Useful for showing proportions

✓ Angle of sector = (Value / Total) × 360°

Types of Pie Chart
(वृत्त आरेख के प्रकार)

I. Simple Pie Chart

(साधारण वृत्त आरेख)

II. Exploded Pie Chart

(विस्फोटित वृत्त आरेख)

III. Multiple Pie Chart

(बहु वृत्त आरेख)

IV. Doughnut Chart

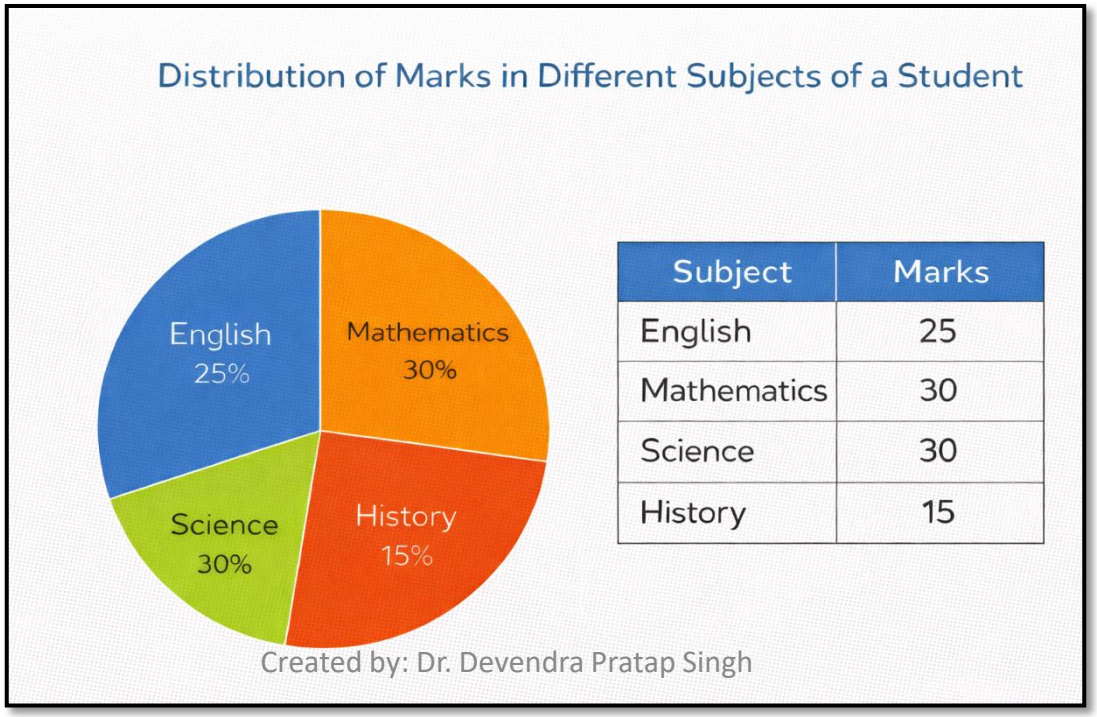
(डोनट आरेख: वृत्ताकार छिद्र आरेख)

Simple Pie Chart (साधारण वृत्त आरेख)

- Simple pie chart is a circular graph that represents **a single set of data**, where the whole circle is divided into sectors, and each sector shows a proportion of the total.
- यह एक गोलाकार आरेख होता है, जिसमें केवल एक डेटा सेट को दर्शाया जाता है। इसमें पूरे वृत्त को विभिन्न खण्डों में विभाजित किया जाता है एवं प्रत्येक खण्ड कुल का एक निश्चित भाग दर्शाता है।

✓ **Uses:** Easy to understand proportions.

✓ **Example:** Distribution of marks in different subjects of a student.

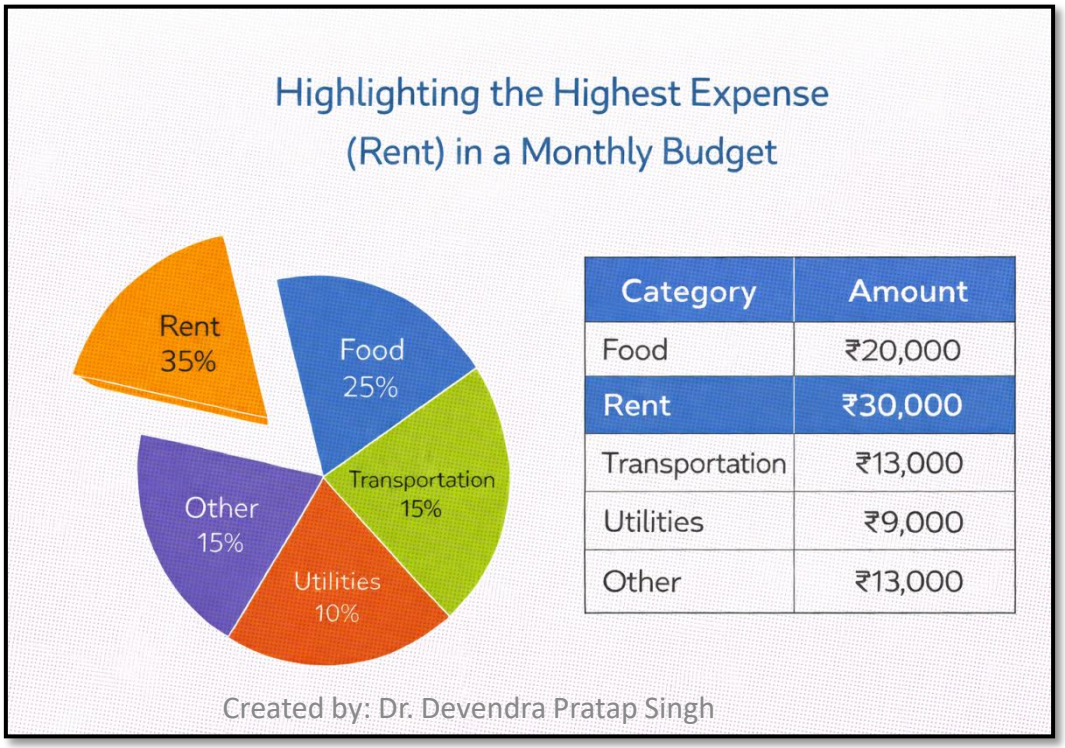


Subject	Marks
English	25
Mathematics	30
Science	30
History	15

Exploded Pie Chart (विस्फोटित वृत्त आरेख)

- Exploded pie chart is a type of pie chart in which **one or more slices are separated (pulled out)** from the main circle to highlight a particular category or value.
- विस्फोटित वृत्त आरेख वह पाई चार्ट है जिसमें एक या अधिक खण्डों को मुख्य वृत्त से अलग (बाहर की ओर खींचकर) दिखाया जाता है, ताकि किसी विशेष भाग को प्रमुखता दी जा सके।

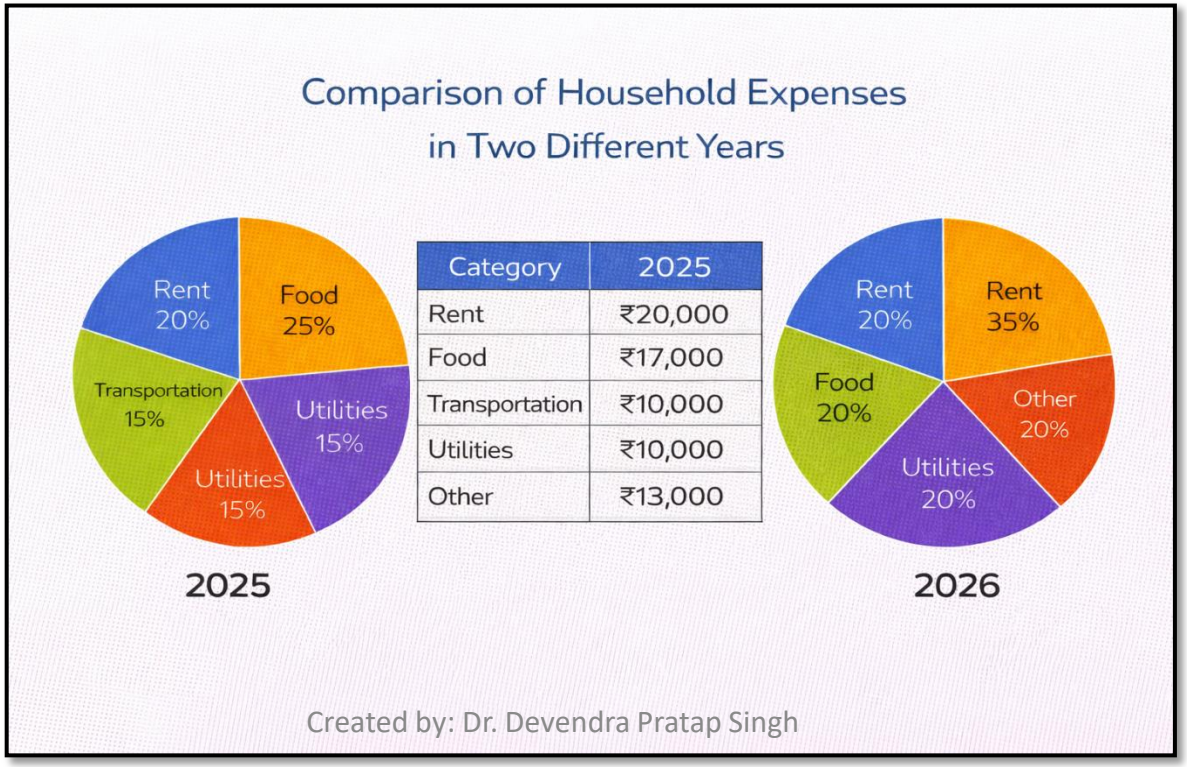
✓ **Key Features**: One or more slices are separated and Highlights important data.
Example: Highlighting the highest expense (rent) in a monthly budget.



Category	Amount
Food	₹20,000
Rent	₹30,000
Transportation	₹13,000
Utilities	₹9,000
Other	₹13,000

Multiple Pie Chart (बहु वृत्त आरेख)

- Multiple pie chart consists of **two or more pie charts shown together**, each representing a different dataset.
- बहु वृत्त आरेख, दो या अधिक पाई चार्ट एक समूह है, जहाँ प्रत्येक चार्ट अलग-अलग डेटा सेट को दर्शाता है।
- **Example:** Comparison of household expenses in two different years.
- **Uses:** Use to compare the composition of data across different groups or time periods.



Doughnut Chart (डोनट आरेख: वृत्ताकार छिद्र आरेख)

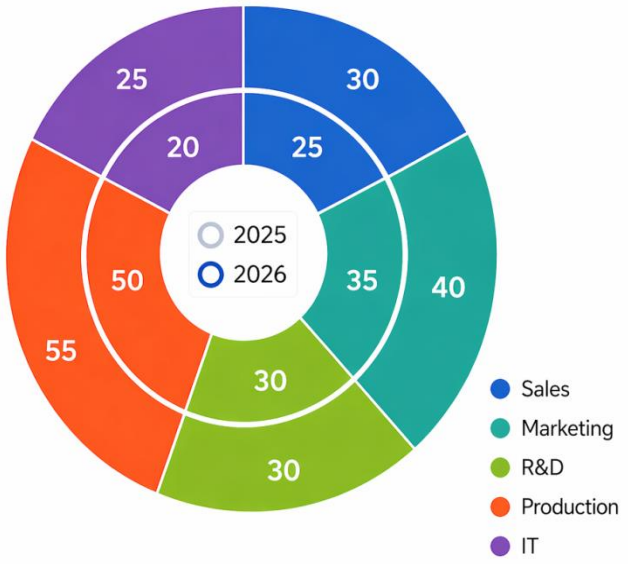
- Doughnut chart is similar to a pie chart but it has a **hole in the centre**. It represents the parts of a whole and can also display more than one data set.
- डोनट आरेख एक ऐसा पाई चार्ट जैसा है जिसके बीच में एक छिद्र (hole) होता है। यह आंकड़े के कुल भाग को दर्शाता है तथा एक से अधिक डेटा सेट भी दिखा सकता है।

✓ **Key Features:** Has a central hole and can show multiple data series (layers).

✓ **Example:** Company sales distribution across departments (with multiple years).

Company Sales Distribution

Department	2025	2026
Sales	25	30
Marketing	35	40
R&D	30	30
Production	50	55
IT	20	25



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4. Histogram (आवृत्ति दंड आरेख)

- Histogram is a type of graph in which **continuous data is divided into class intervals** and their frequencies are represented by **connected rectangles bar.**
- हिस्टोग्राम एक ऐसा ग्राफ है जिसमें सतत डेटा को वर्ग अंतरालों में विभाजित करके उनकी आवृत्तियों को आपस में जुड़े आयत दंडों द्वारा दर्शाया जाता है।

Uses:

✓ To study frequency distribution.

✓ To understand data pattern and shape.

✓ Useful in statistics and research.

Difference Between Histogram and Bar Graph

Basis	Histogram	Bar Graph
Data Type	Continuous Data	Discrete or Categorical Data
Bars	Bars are touching (no gaps)	Bars have gaps between them
Representation	Shows frequency distribution	Shows comparison of categories
Order	Fixed order (class intervals)	Order can be changed
Width of Bars	Represents class interval	Usually equal width
Use	To study distribution and pattern	To compare different groups

Type of Histogram
(हिस्टोग्राम के प्रकार)

1. **Symmetrical Histogram**
   (सममित हिस्टोग्राम)
2. **Positively Skewed Histogram**
   (धनात्मक विकृत हिस्टोग्राम)
3. **Negatively Skewed Histogram**
   (ऋणात्मक विकृत हिस्टोग्राम)
4. **Bimodal Histogram**
   (द्वि-शीर्ष हिस्टोग्राम)
5. **Uniform Histogram**
   (समान हिस्टोग्राम)
6. **Random Histogram**
   (अनियमित हिस्टोग्राम)

Symmetrical Histogram (सममित हिस्टोग्राम)

- Symmetrical histogram is a histogram in which data is equally **distributed on both sides of the centre** and framing a like **Bell Shaped**.
- सममित हिस्टोग्राम वह हिस्टोग्राम है जिसमें डेटा केंद्र के दोनों ओर समान रूप से वितरित होता है एवं इसका आकार घंटी की तरह होता है।

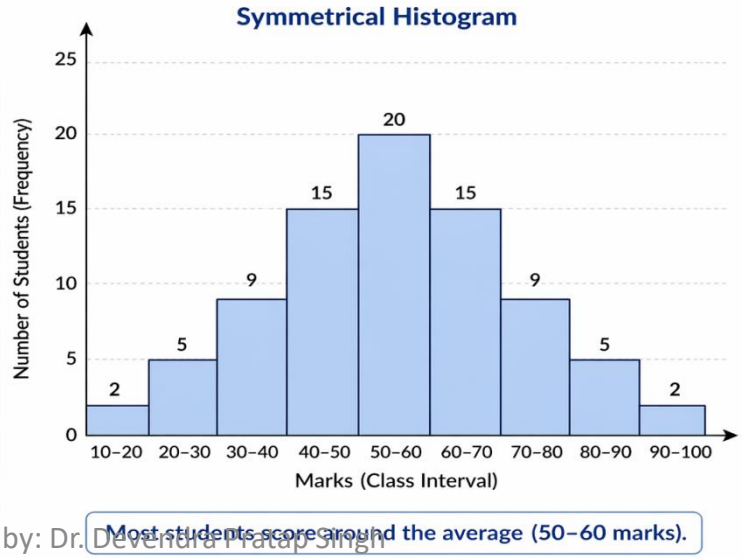
✓ **Key Features:** Left side=Right side (mirror image) and represents normal distribution.

✓ **Example:** Marks of students where most students score around the average.

Symmetrical Histogram

(Marks of Students – Most Students Score Around the Average)

Marks (Class Interval)	Number of Students (Frequency)
10 – 20	2
20 – 30	5
30 – 40	9
40 – 50	15
50 – 60	20
60 – 70	15
70 – 80	9
80 – 90	5
90 – 100	2
Total	**82**



Positively Skewed Histogram (धनात्मक विकृत हिस्टोग्राम)

- Positively skewed histogram is a distribution in which most of the data values are concentrated on the **left side** and a **long tail extends towards the right side**.
- धनात्मक विकृत हिस्टोग्राम वह वितरण है जिसमें अधिकांश डेटा मान बाईं ओर केंद्रित होते हैं एवं एक लंबी पूंछ दाईं ओर फैली होती है।

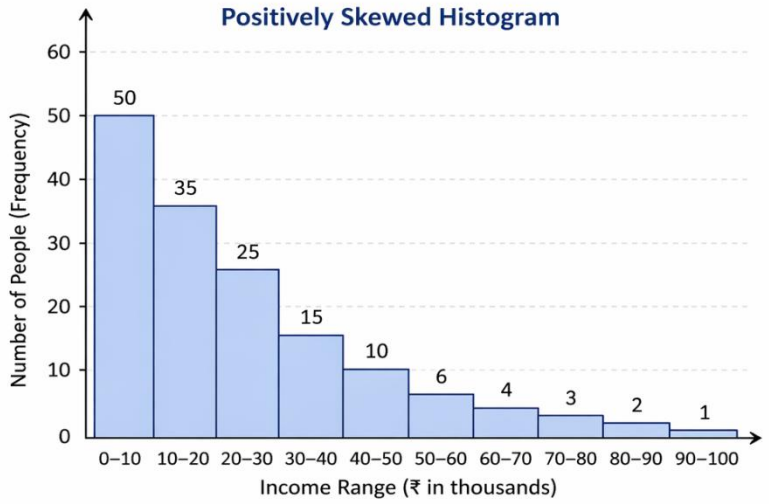
✓ **Key Feature:** Mean > Median > Mode.

✓ **Example:** Income distribution where most people earn less and few are earn very high.

Positively Skewed Histogram

(Income Distribution – Most People Earn Less and Few Earn Very High)

Income Range (₹ in thousands)	Number of People (Frequency)
0 – 10	50
10 – 20	35
20 – 30	25
30 – 40	15
40 – 50	10
50 – 60	6
60 – 70	4
70 – 80	3
80 – 90	2
90 – 100	1
Total	**151**



Key Observations:
- Most people earn in the lower income ranges.
- Frequency decreases as income increases.
- The tail is on the right side – indicating positive skewness.

In Positively Skewed Distribution:
Mean > Median > Mode

Negatively Skewed Histogram (ऋणात्मक विकृत हिस्टोग्राम)

- Negatively skewed histogram is a distribution in which most of the data values are concentrated on the **right side** and a **long tail extends towards the left side**.
- ऋणात्मक विकृत हिस्टोग्राम वह वितरण है जिसमें अधिकांश डेटा मान दाईं ओर केंद्रित होते हैं और एक लंबी पूंछ बाईं ओर फैली होती है।

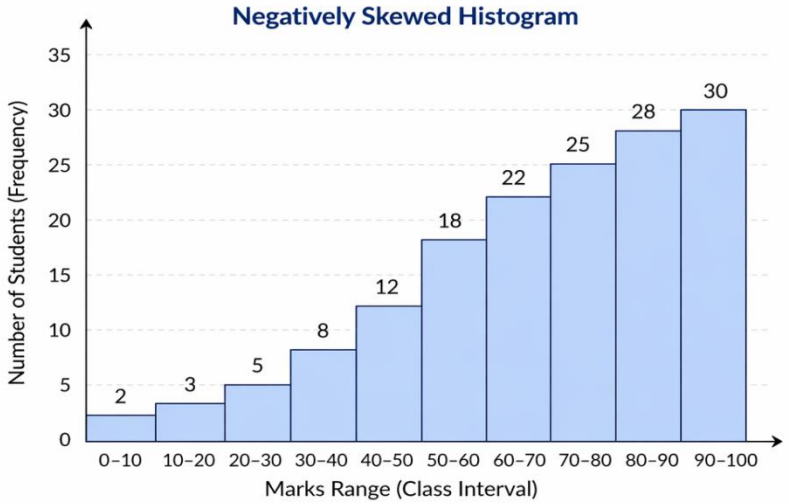
✓ **Key Feature:** Mean < Median < Mode.

✓ **Example:** Exam results where most students score high marks and few are score low.

Negatively Skewed Histogram

(Exam Results – Most Students Score High Marks and Few Score Low)

Marks Range (Class Interval)	Number of Students (Frequency)
0 – 10	2
10 – 20	3
20 – 30	5
30 – 40	8
40 – 50	12
50 – 60	18
60 – 70	22
70 – 80	25
80 – 90	28
90 – 100	30
Total	**153**



Key Observations:
- Most students scored in the higher marks ranges.
- Frequency increases as marks increase.
- The tail is on the left side – indicating negative skewness.

In Negatively Skewed Distribution:
Mean < Median < Mode

Bimodal Histogram (द्वि-शीर्ष हिस्टोग्राम)

- Bimodal histogram is a distribution that has **two distinct peaks**, indicating that the data contains **two different groups or clusters**.
- द्वि-शीर्ष हिस्टोग्राम वह वितरण है जिसमें दो स्पष्ट शिखर होते हैं, जो यह दर्शाते हैं कि डेटा में दो अलग-अलग समूह या क्लस्टर मौजूद हैं।

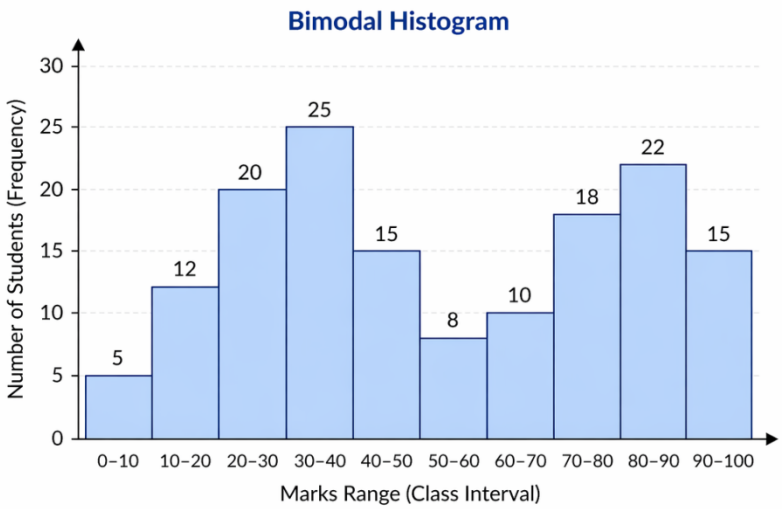
✓ **Key Features:** Not symmetric usually and show mixed population.

✓ **Example:** Marks of students where one group scored low and another scored high.

Bimodal Histogram

(Marks of Students – One Group Scored Low and Another Scored High)

Marks Range (Class Interval)	Number of Students (Frequency)
0 – 10	5
10 – 20	12
20 – 30	20
30 – 40	25
40 – 50	15
50 – 60	8
60 – 70	10
70 – 80	18
80 – 90	22
90 – 100	15
Total	**150**



Key Observations:
- Two distinct peaks are visible.
- First group (low scorers) has more marks between 10–40.
- Second group (high scorers) has more marks between 70–90.
- Indicates two different groups of students.

In Bimodal Distribution:
Two Modes
(Two Peaks in the Histogram)

Uniform Histogram (समान हिस्टोग्राम)

- Uniform histogram is a type of histogram in which **all class intervals have approximately equal frequencies**, resulting in bars of nearly the same height.
- समान हिस्टोग्राम वह हिस्टोग्राम है जिसमें सभी वर्ग अंतरालों की आवृत्ति लगभग समान होती है, जिससे सभी बार लगभग एक समान ऊँचाई के दिखाई देते हैं।

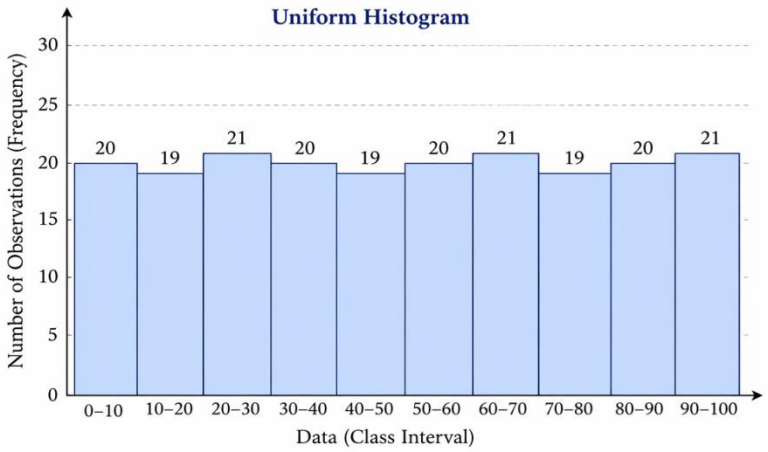
✓ **Key Features:** No clear peak and no skewness.

✓ **Example:** Random distribution where each interval has similar number of observations.

Uniform Histogram

(Random Distribution – Each Interval has Similar Number of Observations)

Data (Class Interval)	Number of Observations (Frequency)
0 – 10	20
10 – 20	19
20 – 30	21
30 – 40	20
40 – 50	19
50 – 60	20
60 – 70	21
70 – 80	19
80 – 90	20
90 – 100	21
Total	**200**



Key Observations:
- All class intervals have nearly equal frequencies.
- Bars are almost of the same height.
- Indicates uniform (equal) distribution of data.

Random Histogram (अनियमित हिस्टोग्राम)

- Random histogram is a histogram in which the data does **not follow any clear pattern or shape**. The frequencies vary irregularly across class intervals.
- अनियमित हिस्टोग्राम वह हिस्टोग्राम है जिसमें डेटा किसी निश्चित पैटर्न या आकार का पालन नहीं करता। विभिन्न वर्गों की आवृत्तियाँ अनियमित रूप से बदलती रहती हैं।

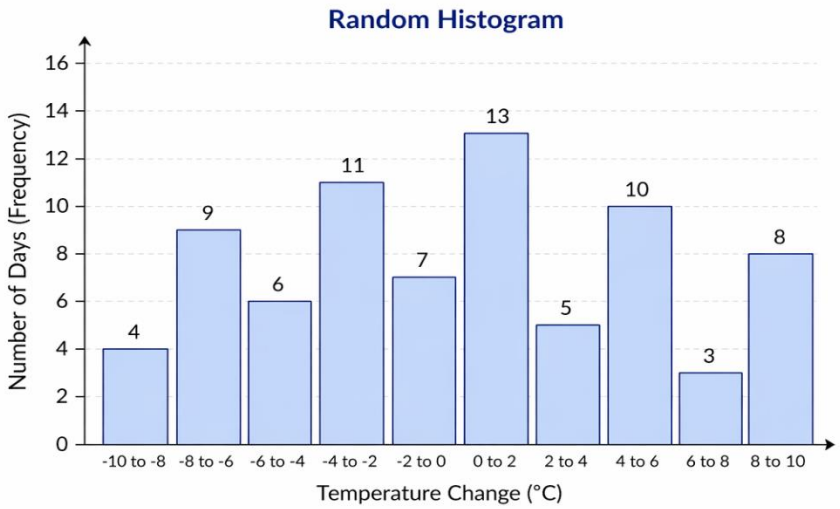
✓ **Key Features:** Bars have uneven heights and no symmetry or skewness pattern.

✓ **Example:** Random data such as daily temperature changes or irregular survey responses.

Random Histogram

(Random Data such as Daily Temperature Changes or Irregular Survey Responses)

Temperature Change (°C)	Number of Days (Frequency)
-10 to -8	4
-8 to -6	9
-6 to -4	6
-4 to -2	11
-2 to 0	7
0 to 2	13
2 to 4	5
4 to 6	10
6 to 8	3
8 to 10	8
Total	**76**



Key Observations:
- There is no fixed pattern in the data.
- Frequencies vary irregularly across intervals.
- Bars have different heights.
- Represents random or irregular distribution.

In Random Distribution:
No specific shape or trend.
Frequencies are uneven and unpredictable.

5. Frequency Polygon (आवृत्ति बहुभुज)

- Frequency polygon is a graph made by plotting **frequencies at the midpoints of class intervals** and joining these points with straight lines to show the distribution of data.
- आवृत्ति बहुभुज एक ऐसा ग्राफ है जिसमें वर्ग अंतरालों के मध्य बिंदुओं पर आवृत्तियाँ प्लॉट की जाती हैं और उन्हें सीधी रेखाओं से जोड़कर डेटा का वितरण दर्शाया जाता है।

Key Features :

✓ Based on class intervals and frequencies.

✓ Uses midpoints of intervals.

✓ Points are connected by straight lines.

✓ Can be drawn with or without histogram.

Difference between Frequency Polygon and Histogram

Basis	Histogram	Frequency Polygon
Representation	Uses rectangular **bars**	Uses **lines** (joined points)
Data plotting	Class intervals on X-axis, frequencies as bar height	Midpoints of class intervals plotted and joined
Appearance	Solid blocks (bars)	Broken line (polygon shape)
Comparison	Difficult to compare multiple datasets	Easy to compare multiple datasets
Visual clarity	More detailed for individual classes	Better for showing overall pattern
Gaps	No gaps between bars	Lines connect points directly

Similarity between Frequency Polygon and Histogram

- Both represent frequency distribution.
- Both use class intervals and frequencies.
- Both help in understanding shape of data.
- Both are used in statistics and research.

How to Make a Frequency Polygon

(Step-by-Step)

- **Step 1: Prepare Data** (Make a table with class intervals and frequencies)
- **Step 2: Find Midpoints** (Midpoint = (Lower limit + Upper limit)÷ 2
- **Step 3: Add Two Extra Classes** (Add one extra class interval at both ends with zero frequency)

  (To make the frequency polygon touch the X-axis and form a complete and closed shape)
- **Step 4: Draw the Axes** (X-axis⟶ Midpoints and Y-axis⟶ Frequencies)
- **Step 5: Plot the Points** (Plot each midpoint against its frequency)
- **Step 6: Join the Points** (Join all plotted points using straight lines and ensure that first and last points touch the X-axis)

Type of Frequency Polygon

(आवृत्ति बहुभुज के प्रकार)

I. Simple Frequency Polygon

(साधारण आवृत्ति बहुभुज)

II. Multiple Frequency Polygon

(बहु आवृत्ति बहुभुज)

III. Superimposed Frequency Polygon

(अध्यारोपित आवृत्ति बहुभुज)

Simple Frequency Polygon (साधारण आवृत्ति बहुभुज)

- Simple frequency polygon is a **single set of data** graph by plotting frequencies at the **midpoints of class intervals** and joining them with straight lines.
- साधारण आवृत्ति बहुभुज एक ही डेटा सेट पर आधारित ग्राफ है जिसमें वर्ग अंतरालों के मध्य बिंदुओं पर आवृत्तियाँ प्लॉट करके उन्हें सीधी रेखाओं से जोड़ा जाता है।

✓ **Key Features:** Represents one dataset only and Uses midpoints of class intervals.

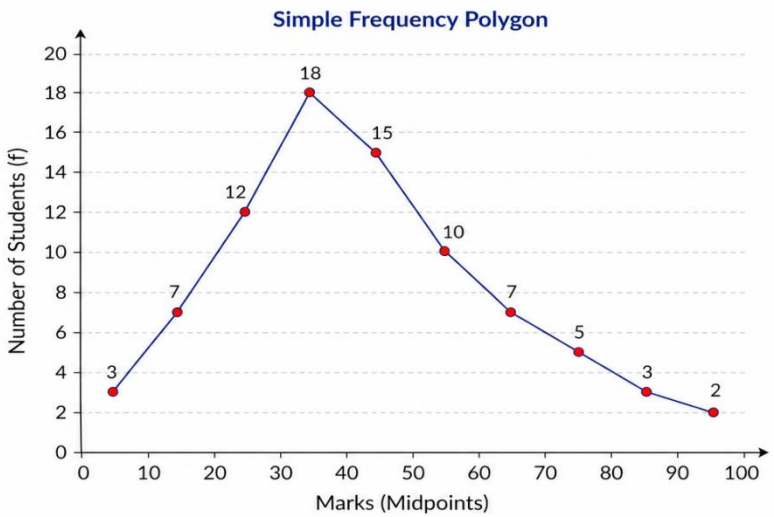
✓ **Example:** Marks of students in one class grouped into intervals

Simple Frequency Polygon

(Marks of Students in One Class Grouped into Intervals)

Data Table

Marks (Class Interval)	Midpoint (X)	Number of Students (f)
0 – 10	5	3
10 – 20	15	7
20 – 30	25	12
30 – 40	35	18
40 – 50	45	15
50 – 60	55	10
60 – 70	65	7
70 – 80	75	5
80 – 90	85	3
90 – 100	95	2
Total		**82**



Key Points:
- Midpoints of each class interval are taken on the X-axis.
- Corresponding frequencies are taken on the Y-axis.
- Points are joined with straight lines to form the frequency polygon.
- To close the polygon, join the first and last points to the X-axis with zero frequency.

Multiple Frequency Polygon (बहु आवृत्ति बहुभुज)

- Multiple frequency polygon is a graph in which **two or more frequency polygons are drawn on the same axes** to compare different datasets.
- बहु आवृत्ति बहुभुज वह ग्राफ है जिसमें दो या दो से अधिक आवृत्ति बहुभुज एक ही अक्षों पर बनाए जाते हैं, ताकि विभिन्न डेटा सेट की तुलना की जा सके।

✓ **Key Features:** Represents multiple datasets and Drawn on the same graph with different colour lines.

✓ **Example:** Marks of boys and girls plotted on the same graph.

Multiple Frequency Polygon

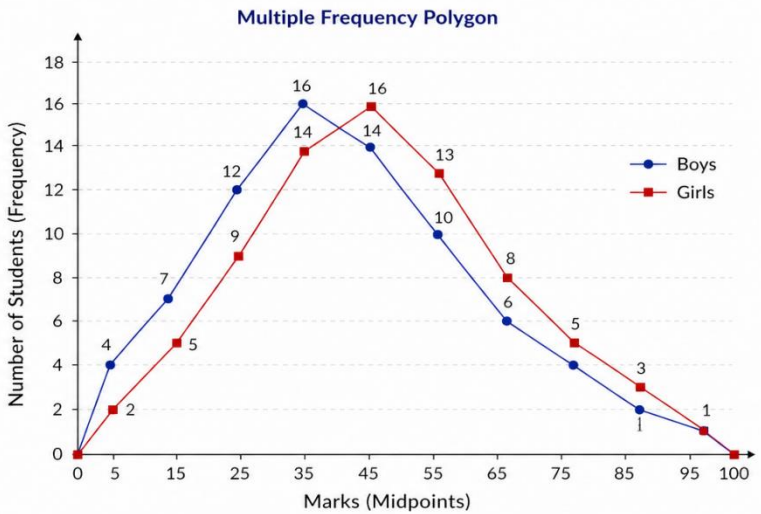
(Marks of Boys and Girls Plotted on the Same Graph)

Marks of Boys

Marks (Class Interval)	Midpoint (X)	Number of Boys (Frequency)
0 – 10	5	4
10 – 20	15	7
20 – 30	25	12
30 – 40	35	16
40 – 50	45	14
50 – 60	55	10
60 – 70	65	6
70 – 80	75	4
80 – 90	85	2
90 – 100	95	1
Total		76

Marks of Girls

Marks (Class Interval)	Midpoint (X)	Number of Girls (Frequency)
0 – 10	5	2
10 – 20	15	5
20 – 30	25	9
30 – 40	35	14
40 – 50	45	16
50 – 60	55	13
60 – 70	65	8
70 – 80	75	5
80 – 90	85	3
90 – 100	95	1
Total		76



Key Points:
- Class marks (midpoints) are taken on the X-axis.
- Frequencies are taken on the Y-axis.
- Two frequency polygons are drawn on the same graph.
- Different colors/symbols are used for easy comparison.

Observation:
- In lower marks (0–30), boys have more frequency.
- In higher marks (40 and above), girls have more frequency.
- Both reach the same total number of students (76).

Superimposed Frequency Polygon (अध्यारोपित आवृत्ति बहुभुज)

- Superimposed frequency polygon is a graph where **one or more frequency polygons are drawn on the same axes, usually over a histogram** or overlapping each other to compare distributions clearly.
- अध्यारोपित आवृत्ति बहुभुज वह ग्राफ है जिसमें एक या अधिक आवृत्ति बहुभुज एक ही अक्षों पर, प्रायः हिस्टोग्राम के ऊपर या एक-दूसरे के ऊपर बनाए जाते हैं, ताकि वितरण की स्पष्ट तुलना की जा सके।

✓ **Key Features:** Drawn over a histogram or overlapping each other and Uses same axes (X and Y).

✓ **Example:** A histogram of marks with a frequency polygon drawn over it.

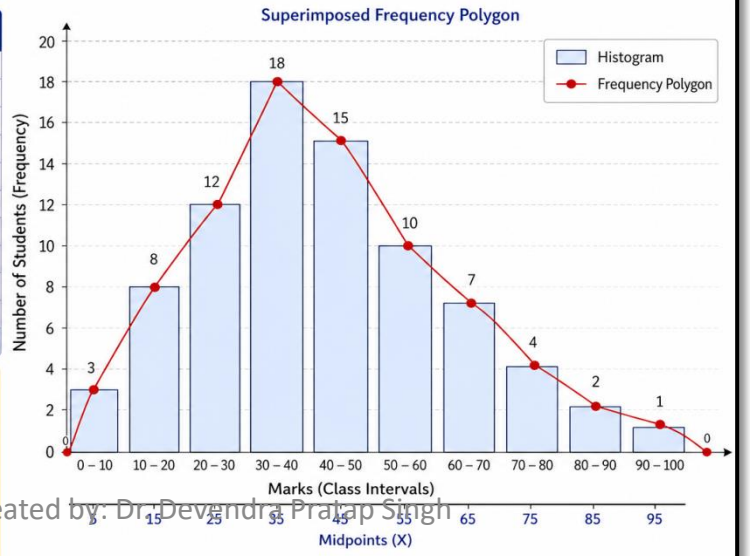
Superimposed Frequency Polygon

Histogram of Marks with a Frequency Polygon Drawn Over It

Data Table

Marks (Class Interval)	No. of Students (Frequency, f)	Midpoint (X)
0 – 10	3	5
10 – 20	8	15
20 – 30	12	25
30 – 40	18	35
40 – 50	15	45
50 – 60	10	55
60 – 70	7	65
70 – 80	4	75
80 – 90	2	85
90 – 100	1	95
Total	**80**	

Key Points:
- Class intervals are shown on the X-axis.
- Frequencies are shown on the Y-axis.
- Histogram is drawn using class intervals and frequencies.
- Frequency polygon is drawn by joining the midpoints of the top of each bar.
- The polygon touches the X-axis at both ends (before the first class and after the last class) with zero frequency.



6. Frequency Curve (आवृत्ति वक्र)

- Frequency curve is a **smooth, continuous curve** that represents the **distribution of frequencies** of a dataset. It is obtained by smoothing a frequency polygon.
- आवृत्ति वक्र एक smooth एवं continuous वक्र होता है जो डेटा की आवृत्तियों के वितरण को दर्शाता है। यह आवृत्ति बहुभुज को smooth करने पर प्राप्त होता है।

Key Features:

✓ Smooth and continuous curve.

✓ No sharp angles.

Uses:

✓ To understand shape of distribution.

✓ To study normal distribution.

✓ Useful in statistics and research.

Difference between Frequency Polygon and Frequency Curve

Basis	Frequency Polygon	Frequency Curve
Shape	Made of straight line segments	Smooth continuous curve
Nature	Angular (sharp turns)	Smooth (no sharp angles)
Formation	Joining plotted midpoints with lines	Smoothing the frequency polygon
Accuracy	Less smooth and more rigid	More realistic and representable
Use	Simple representation of data	Shows overall pattern clearly

Frequency Polygon and Frequency Curve of Same Data

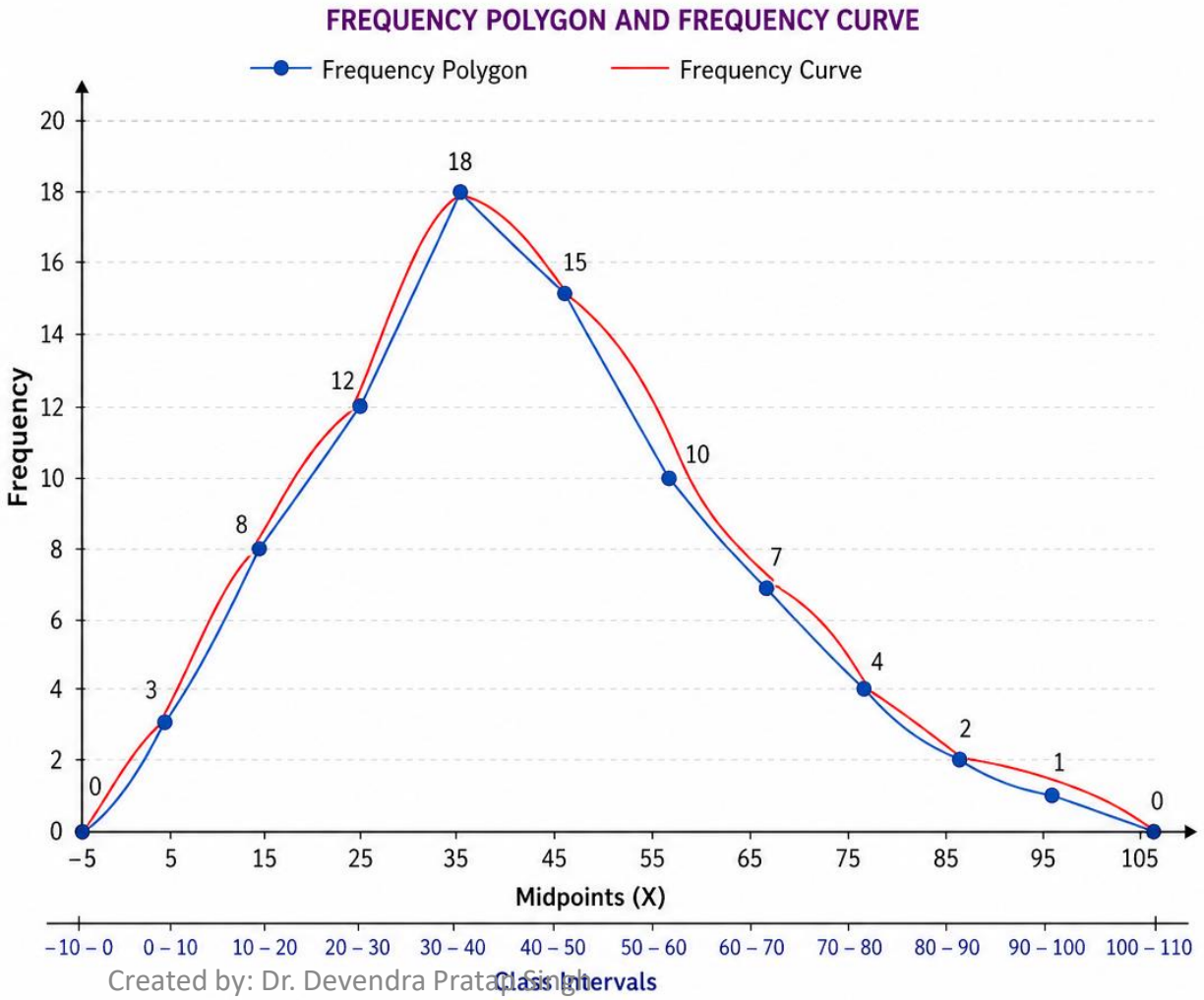
FREQUENCY POLYGON AND FREQUENCY CURVE
(For the Same Data)

DATA TABLE

Class Interval	Frequency (f)	Midpoint (X)
0 – 10	3	5
10 – 20	8	15
20 – 30	12	25
30 – 40	18	35
40 – 50	15	45
50 – 60	10	55
60 – 70	7	65
70 – 80	4	75
80 – 90	2	85
90 – 100	1	95
–10 – 0	0	–5
100 – 110	0	105

KEY POINTS

- Extra class intervals (–10 – 0 and 100 – 110) with zero frequency are added.
- Midpoints are plotted on the X-axis.
- Frequencies are plotted on the Y-axis.
- Frequency Polygon is formed by joining the midpoints with straight lines.
- Frequency Curve is a smooth curve drawn through the points of the polygon.



7. Ogive: Cumulative Frequency Curve

(ओजाइव: संचयी आवृत्ति वक्र)

- Ogive is a graph that represents the **cumulative frequency distribution** of data. It is drawn by plotting cumulative frequencies against class boundaries and joining the points with a smooth curve.
- तोरण वह ग्राफ है जिसमें संचयी आवृत्ति को वर्ग सीमाओं के साथ प्लॉट किया जाता है एवं बिंदुओं को smooth लाइन से मिलाकर बनाया जाता है।

Key Features:

✓ Based on cumulative frequency and a Smooth curve.

Uses: (Statistical Output of Ogive)

✓ To find Median and Quartiles.

✓ To find Percentiles and Interquartile Range (IQR).

✓ To Make Comparison of Distributions.

Type of Ogive

(प्रकार)

I. Less Than Ogive: Cumulative Addition

(कम से कम तोरण)

II. More Than Ogive: Cumulative Subtraction

(अधिक से अधिक तोरण)

Less Than Ogive (कम से कम तोरण)

- Less than ogive is a cumulative frequency curve where cumulative frequencies are plotted against the **upper class boundaries**.
- कम से कम वक्र, वह वक्र है जिसमें संचयी आवृत्तियों को ऊपरी वर्ग सीमाओं के साथ प्लॉट किया जाता है।

Key Point:

✓ Curve moves upward (increasing order).

Uses:

✓ To find Median and Quartiles.

✓ To Make Comparison of Distributions.

Creation Process of Less Than Ogive

(कम से कम तोरण निर्माण की प्रक्रिया)

- **Step 1:** Take upper class boundaries.
- **Step 2:** Add frequencies cumulatively from top to bottom.
- **Step 3:** Plot the upper class boundary value on X-Axis and related cumulative frequency value on Y-Axis.

Class Interval	Upper Class Boundary	Frequency	Cumulative Frequency (Less Than)
0 – 10	10	5	5
10 – 20	20	9	5 + 9 = 14
20 – 30	30	14	14 + 14 = 28
30 – 40	40	8	28 + 8 = 36
40 – 50	50	4	36 + 4 = 40

Less Than Ogive

LESS THAN OGIVE

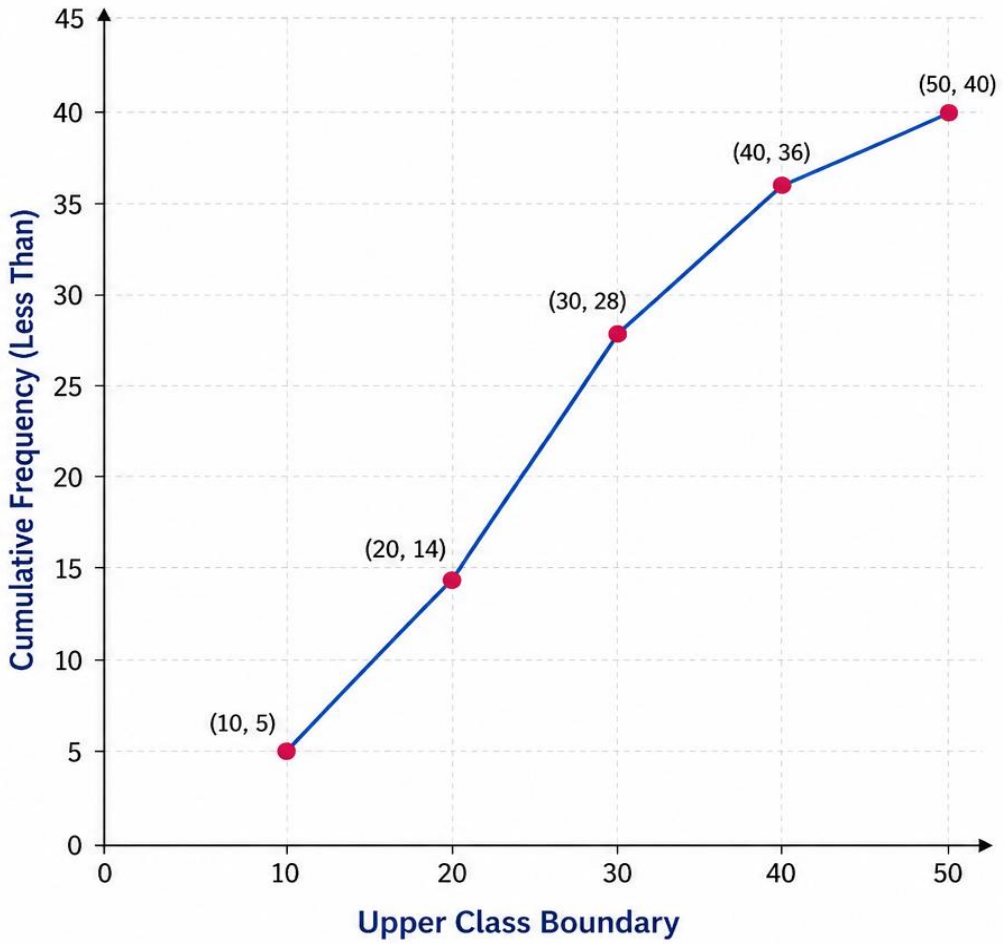
Data Table

Class Interval	Upper Class Boundary	Frequency (f)	Cumulative Frequency (Less Than)
0 – 10	10	5	5
10 – 20	20	9	14
20 – 30	30	14	28
30 – 40	40	8	36
40 – 50	50	4	40

Points plotted

(Upper Class Boundary, Cumulative Frequency)

- (10, 5)
- (20, 14)
- (30, 28)
- (40, 36)
- (50, 40)



More Than Ogive (अधिक से अधिक तोरण)

- More than ogive is a cumulative frequency curve in which cumulative frequencies are plotted against the **lower class boundaries**.
- अधिक से अधिक वक्र वह संचयी आवृत्ति वक्र है जिसमें संचयी आवृत्तियाँ निचली वर्ग सीमाओं के साथ प्लॉट की जाती हैं

Key Features :

- ✓ Curve is downward (decreasing).
- ✓ Showing how many observations are more than a given value.
- ✓ Starts from total frequency (N) and ends at zero frequency.

Uses:

- ✓ To find Percentiles and Interquartile Range (IQR).
- ✓ To Make Comparison of Distributions.

Creation Process of More Than Ogive

(अधिक से अधिक तोरण निर्माण की प्रक्रिया)

- **Step 1:** Take lower class boundaries.
- **Step 2:** Add frequencies cumulatively from bottom to top.
- **Step 3:** Plot the lower class boundary value on X-Axis and related cumulative frequency value on Y-Axis.

Class Interval	Lower Class Boundary	Frequency	Cumulative Frequency (More Than)
0 – 10	0	5	40
10 – 20	10	9	40 – 5 = 35
20 – 30	20	14	35 – 9 = 26
30 – 40	30	8	26 – 14 = 12
40 – 50	40	4	12 – 8 = 4

More Than Ogive

MORE THAN OGIVE

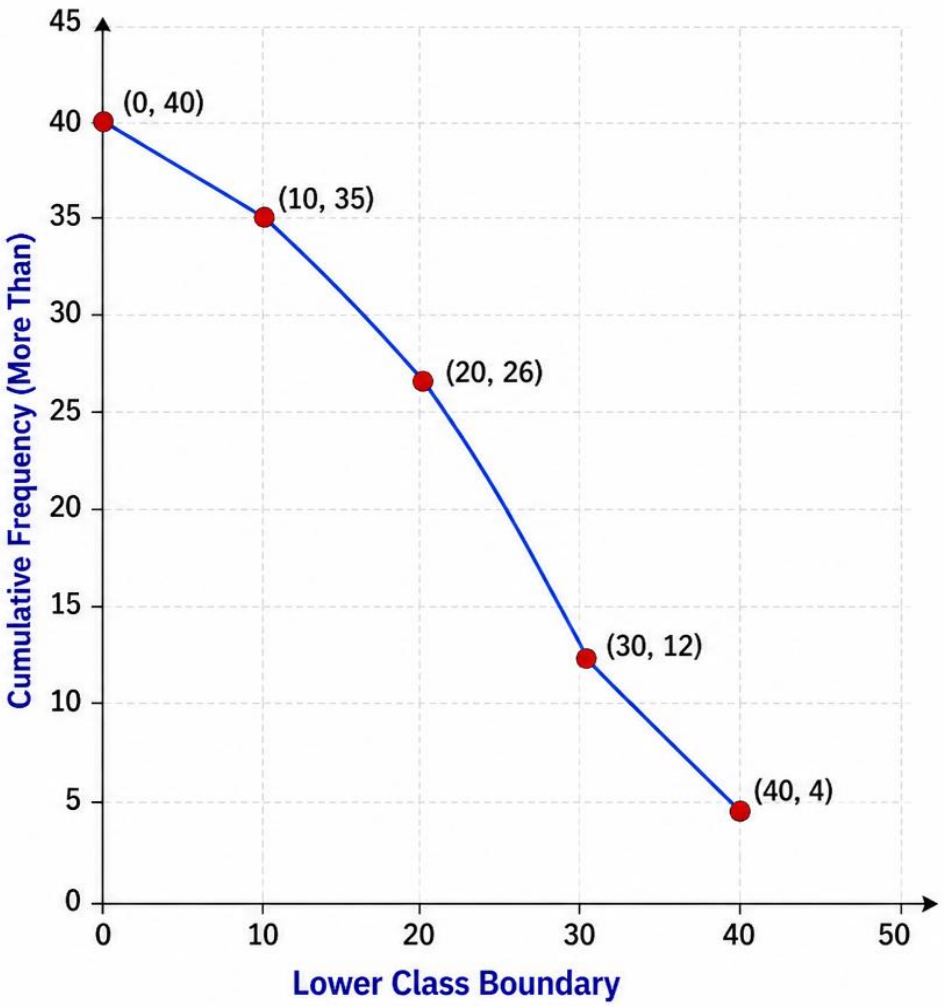
Data Table

Class Interval	Lower Class Boundary	Frequency (f)	Cumulative Frequency (More Than)
0 – 10	0	5	40
10 – 20	10	9	40 – 5 = 35
20 – 30	20	14	35 – 9 = 26
30 – 40	30	8	26 – 14 = 12
40 – 50	40	4	12 – 8 = 4

Points to be plotted

(Lower Class Boundary, Cumulative Frequency More Than)

(0, 40), (10, 35), (20, 26), (30, 12), (40, 4)



Learning Outcomes
(अधिगम परिणाम)

After learned this unit, the learner will be able:

- **To explain** the meaning and importance of graphical presentation of data.
- **To describe** the contributions of René Descartes and William Playfair in the development of graphs.
- **To identify** and label the X-axis, Y-axis, origin, quadrants, and scales correctly.
- **To use** suitable scales and broken-axis symbols properly in graphs.
- **To draw** and interpret bar graphs, line graphs, pie charts, histograms, frequency polygons, frequency curves, and ogives.

Learning Outcomes

(अधिगम परिणाम)

After learned this unit, the learner will be able:

- **To differentiate** between histograms and bar graphs, as well as frequency polygons and frequency curves.
- **To construct** less than ogive and more than ogive from cumulative frequency tables.
- **To identify** and interpret positively skewed, negatively skewed, symmetrical, bimodal, uniform, and random histograms.
- **To analyze** graphical data and draw meaningful conclusions from it.
- **To apply** graphical presentation methods in statistics, educational research, and practical life situations.

Self Assessment Questions

1. **Who developed the Cartesian Coordinate System?**
   A. William Playfair **B.** John Tukey
   C. René Descartes **D.** Florence Nightingale

2. **Which graph is used to show parts of a whole?**
   A. Histogram **B.** Pie Chart
   C. Ogive **D.** Frequency Polygon

3. **The horizontal axis in a graph is called:**
   A. Ordinate **B.** Origin
   C. Abscissa **D.** Quadrant

4. **The point where X-axis and Y-axis intersect is called:**
   A. Midpoint **B.** Origin
   C. Boundary **D.** Scale

5. **Which graph uses adjacent rectangles?**
   A. Bar Graph **B.** Pie Chart
   C. Histogram **D.** Ogive

6. **A frequency polygon is formed by joining:**
   A. Class intervals **B.** Midpoints
   C. Frequencies only **D.** Angles

7. **Which curve is smooth and continuous?**
   A. Frequency Curve **B.** Bar Graph
   C. Histogram **D.** Pie Chart

8. **Less than ogive is drawn using:**
   A. Lower class boundary **B.** Midpoints
   C. Upper class boundary **D.** Angles

Self Assessment Questions

9. **More than ogive generally forms:**
   A. Increasing curve **B.** Decreasing curve
   C. Circular graph **D.** Straight line

10. **Which graph is best for comparing categories?**
    A. Bar Graph **B.** Histogram
    C. Frequency Curve **D.** Ogive

11. **A histogram represents:**
    A. Discrete data **B.** Continuous data
    C. Percentage only **D.** Ratio only

12. **Which histogram has one central peak and symmetry?**
    A. Random **B.** Bimodal
    C. Symmetrical **D.** Negatively skewed

13. **In a positively skewed histogram, the tail extends toward:**
    A. Left side **B.** Right side
    C. Centre **D.** Bottom

14. **Which symbol shows a broken scale on axis?**
    A. × **B.** //
    C. + **D.** ○

15. **Pie chart values are represented in:**
    A. Angles **B.** Length
    C. Frequency only **D.** Width

16. **Who introduced bar graph and line graph?**
    A. René Descartes **B.** William Playfair
    C. John Tukey **D.** Pearson

Self Assessment Questions

17. **Which graph helps to find median graphically?**

A. Pie Chart **B.** Histogram

C. Ogive D. Line Graph

18. **Frequency polygon uses:**

A. Smooth curves **B.** Straight lines

C. Angles only **D.** Circles

19. **Which graph may become misleading due to distorted scale?**

A. Frequency Polygon **B.** Gee-Whiz Graph

C. Pie Chart **D.** Histogram

20. **Which graph is useful for showing trends over time?**

A. Line Graph **B.** Pie Chart

C. Ogive **D.** Histogram

Answer Key

1-C	2-B	3-C	4-B	5-C
6-B	7-A	8-C	9-B	10-A
11-B	12-C	13-B	14-B	15-A
16-B	17-C	18-B	19-B	20-A

Thank You!

Thank you for your time, attention,
and valuable support.