Convergence of Newton's Method

Given f(x)=0

f'(x) != 0 and f''(x) is continuous on the close finite interval [a,b]

Condition:-

1. f(a).f(b)<0
2. f''(x) is either >=0 or <=0 for all x belongs to [a,b]
3. At the end points a,b

(|f(a)| / |f'(a)|) < b-a

and

(|f(b)| / |f'(b)|) < b-a

Then Newton's method converges to the unique solution [a,b]

Q. 2x^3+5x^2+5x+3=0 [-2,-1] converges=?

f(x)= 2x^3+5x^2+5x+3

f'(x)=6x^2+10x+5

f''(x)=12x+10

f(a)=f(-2)=2(-2)^3+5(-2)^2+5(-2)+3=-16+20-10+3=-3

f'(a)=f'(-2)=9

f(b)=f(-1)=1

f'(b)=f'(-1)=1

(|f(a)| / |f'(a)|) < b-a

=> 3/9 < -1 -(-2)

=> 1/3 < 1 .....true

(|f(b)| / |f'(b)|) < b-a

=> 1/1 < 1

1 < 1 .....false

Then Newton's method does not converges to the unique solution [-2,-1].