FRAME OF REFERENCE

B.Sc. III (Paper I)

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Frame of Reference:

If we imagine a coordinate system attached to a rigid body and we describe the position of any particle in space relative to it, then such a coordinate system is called frame of reference.

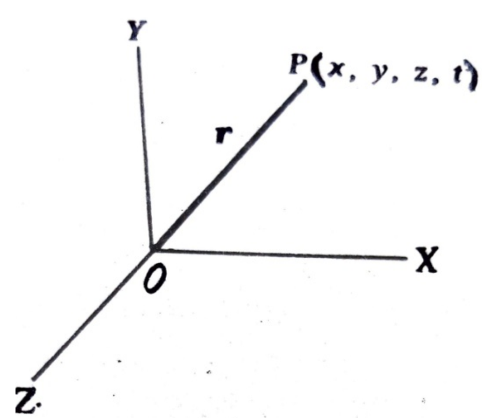
The simplest frame of reference is a Cartesian coordinate system. In this system the position of a particle at any point is given by three coordinates (x, y, z). Now at any instant, the position vector $\vec{r}$ drawn from the origin to the particle, is given by

$$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$$

We have to know position at different instant of time, i.e., the position coordinate $\vec{r}$ should be expressed as function of time. Then the velocity and acceleration of the particle will be given by

$$\vec{v} = \frac{d\vec{r}}{d\vec{t}} \; and \; \vec{a} = \frac{d^2\vec{r}}{dt^2}$$

In general, for the location of an event in a frame of reference we require its position and time of occurrence and therefore for this purpose four coordinates (x, y, z, t) are required. The reference system, employed for this purpose, is called *space-time frame of reference*. The motion of a body is described differently, depending on the frame of reference with respect to which description is given. But in general, that frame of reference is employed in which the motions are easy to describe.



Inertial Frame of Reference:

Those unaccelerated frame of reference, in which Newton's first and second law hold, are called *inertial frame*. Thus, in an inertial frame, if a body is not experiencing any external force, its acceleration $\vec{a}$ is given by

$$\vec{a} = \frac{d^2\vec{r}}{dt^2} = 0 \qquad\qquad (\because\ \vec{F} = m\vec{a} = 0)$$

Let us consider an inertial frame S and another frame S', which is moving with constant velocity $\vec{v}$, relative to S. Initially (at t = 0) if the position of the origins of the two frames coincide, then in the two frames, the position vectors of any particle P at any instant t can be related by following expressions:

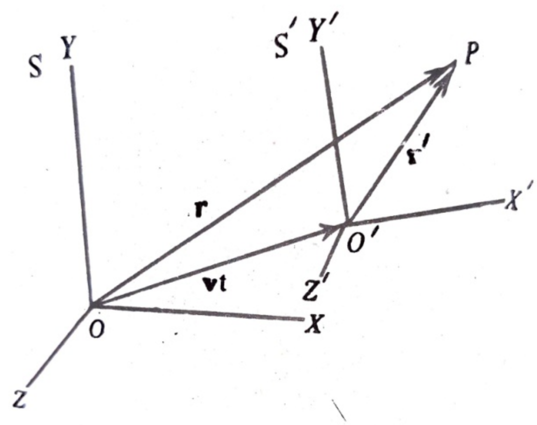
$$r = \overrightarrow{OO'} + \overrightarrow{r'} \text{ or } \vec{r} = \vec{v}t + \overrightarrow{r'}$$

$$\text{or } \overrightarrow{r'} = \vec{r} - \vec{v}t$$

As $\vec{v}$ is constant, $\therefore \frac{d^2r'}{dt^2} = \frac{d^2r}{dt^2}$ $or\ a' = a$

That is, a particle experiences the same acceleration in two frames, which are moving with constant relative velocity. Now, if the acceleration of the particle in frame S is zero, its acceleration in S' is also zero. But S is an inertial frame, S' should also be inertial. <u>Thus, we conclude that, if a frame is an inertial frame, then all those frames which are moving with constant velocity relative to the first frame are also inertial.</u>

Necessarily, the inertial frames are unaccelerated frames because if frame is accelerated, a particle, whose velocity is constant, will appear accelerated in this frame. According to Newton the absolute space represents that system of reference relative to which every motion should be measured. Experience tells that 'fixed stars' are approximately stationary relative to the absolute space because a body sufficiently far removed from celestial matter always moves with a uniform velocity relative to the fixed stars. Thus this absolute system is an frame of reference. It is also clear that the law of inertia is valid in all those reference systems, which are moving with constant velocity relative to the absolute system.



If a frame is fixed on the earth, this frame will not be inertial because earth is spinning about its axis and simultaneously it is moving in its orbit round the sun. For many purposes the earth is fairly good approximation in an inertial frame, but due to the rotation of the earth, a particle at rest on the surface of the earth experiences centripetal acceleration which cannot be entirely neglected for all purposes.

Galilean Transformation:

A point or a particle at any instant, in space has different coordinates in different reference systems. The equations which provide the relationship between the coordinates of two reference systems are called **transformation equations.**

We know that a frame S', which is moving with constant velocity $\vec{v}$ relative to an inertial frame S, is itself inertial.

In view of convenience, if we assume (i) that the origins of the two frames coincide at t=0, (ii) that the coordinate axix of the second frame are parallel to the first and (iii) that the velocity of the second frame relative to the first is $\vec{v}$ along x-axis, then the position vectors of a particle at sny instant t are related by the equation

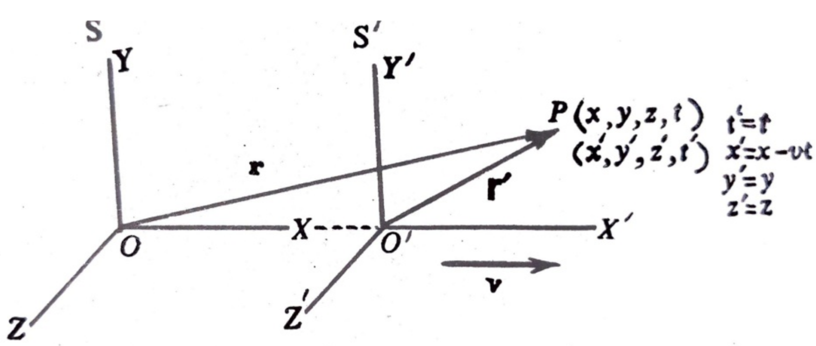
$$\overrightarrow{r'} = \vec{r} - \vec{v}t \qquad (1)$$

In x, y, z the equation (1) is written as:

$$x' = x - \vec{v}t; y' = y - \vec{v}t; z' = z \qquad (2)$$

Equation (1) or (2) expresses the transformation of coordinates from one inertial frame to another and hence they are referred as **Galilean transformations.** The form of eq. (1) or (2) depends on the relative motion of two frames and on the nature of space and time. It is assumed that the time t is independent of any particular frame of reference, i.e., if t and t' be the time recorded by the observers O and O' (figure) of an event occurring at P, then t=t'. If we add this in above eq. (2), then the Galilean transformation equations are:

$$x' = x - \vec{v}t; y' = y - \vec{v}t; z' = z; t' = t \qquad (3)$$



If a rod has length L in the frame S with the end coordinates (x_1, y_1, z_1) and (x_2, y_2, z_2), then

$$L = \sqrt{(x_2 - x_1)^2 + \ (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

If at the same time the end coordinates of the rod in frame S', are (x'_1, y'_1, z'_1) and (x'_2, y'_2, z'_2), then length

$$L' = \sqrt{(x'_2 - x'_1)^2 + \ (y'_2 - y'_1)^2 + (z'_2 - z'_1)^2}$$

But from any time t, from eq. (3), we have

$$x'_2 - x'_1 = \ x_2 - x_1; \ y'_2 - y'_1 = y_2 - y_1; \ z'_2 - z'_1 = z_2 - z_1$$

$$L = L' \qquad (4)$$

Thus, <u>the length of distance between two points is invariant under Galilean transformation.</u>

Now differentiating, eq. (1) w.r.t. time, we get

$$\frac{d\vec{r}}{dt} = \vec{v} + \frac{d\vec{r'}}{dt} = \vec{v} + \frac{d\vec{r'}}{dt'} \qquad \text{(as t=t')}$$

or

$$\vec{V} = \ \vec{v} + \vec{V'} \qquad (5)$$

where $\vec{V}$ and $\vec{V'}$ are the observed velocities in S and S' frames respectively.

Equation (5) transforms the velocity of a particle from one frame to another and is known as *Galilean law of addition of velocities*.

<u>The Hypothesis of Galilean Invariance – Principle of Relativity:</u>

The hypothesis of Galilean invariance is based on experimental observations and can be states as follows:

"<u>The basis law of physics are identical in all reference systems which moves with uniform velocity with respect to one another</u>"

According to this hypothesis, if a windowless spaceship is moving woth uniform speed relative to the fixed stars, then all experiments performed in spaceship and all the phenomena in the spaceship will appear the same to an observer confined to it as if the spaceship were stationary. Only by looking though a window, the observer can tell that he is in uniform motion relative to the stars. Thus, the absolute velocity has no physical meaning only the velocity relative to other objects is meaningful.

Since any two inertial systems, which are moving with constant relative velocity, can be connected by Galilean transformations, we can modify the hypothesis of Galilean invariance by giving the following statements:

"The basis law of Physics are ivariant in form in two reference systems connected by Galilean transformations."

Non-inertial Frames and fictitious force:

Those frame of reference in which Newton's law of inertia does not hold are called non-inertial frames. All the accelerated and rotating frames are the non-inertial frame of reference.

Such as force, which does not really act on the particle bu appear due to the acceleration of the frame is called a fictitious force.

Foucault's Pendulum:

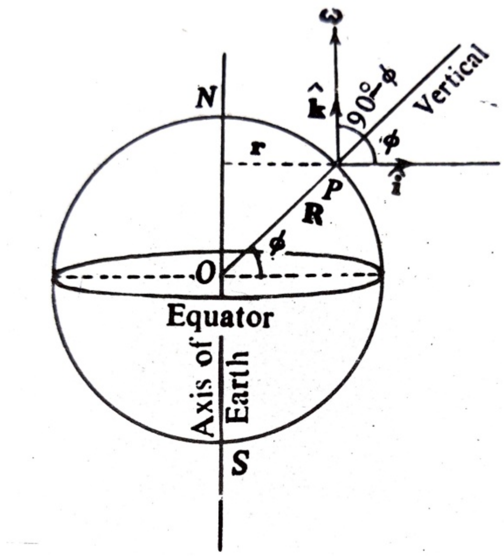
Experimentally this fact, that earth rotates about its axis and so the frame attached to it is not an inertial frame, is demonstrated by Foucault's pendulum. The experiment was first performed by Foucault in 1851.

Foucault's pendulum is similar to the simple pendulum in which very heavy bob is suspended by means of a long strong wire. Thus, if it is once vibrated, it vibrates continuously large time with a large period (=17 sec). Foucault took in his experiment a bob of 28 kg mass and a wire of 70 meter length. The upper end of wire is attached to a rigid support in such a way that the pendulum may vibrates with equal freedom in any direction.

If Foucault's pendulum is allowed to oscillate in the northern hemisphere, its plane of oscillation rotates from east to west (or clockwise if seen from above). It is possible that the point of support might have turned the plane of vibration. The only possibility is that the floor, i.e., the earth under the pendulum may be rotating. If such a pendulum is imagined to vibrate at the north pole of the earth, the plane of oscillation will remain fixed in an inertial frame or solar reference frame. But the earth under the pendulum is rotating once every 24 hours, therefore an observer on the earth will see that the plane of oscillation is turning from east to west, opposite to earth's rotation.

At any other latitude ϕ, the angular velocity ω can be resolved into two components of vertical component $\omega \sin\phi$ and horizontal component $\omega \cos\phi$ in the north-south direction. Evidently the later component will not have any perceptible effect on the pendulum. The vertical component will make the plane of its oscillation rotate with an angular velocity $\omega \sin\phi$. Hence the time of one rotation of the plane of oscillation will be given by

$$T = \frac{2\pi}{\omega \sin\emptyset}$$



As the earth rotates once in 24 hours with an angular velocity ω,

$$24\ hours\ = \frac{2\pi}{\omega}$$

Hence
$$T = \frac{24\ hours}{sin\emptyset}$$

This rotation will be clockwise in the northern hemisphere and opposite to it in southern hemisphere. In fact, this rotation, we seen by the observer on the earth is due to Coriolis force.

<u>Coriolis force is a fictitious force which act on a particle on if it is in motion with respect to the rotating force.</u>

References:

1. Mechanics by J.C. Upadhyaya
2. Mechanics by D.S. Mathur