ZONE PLATE

B.Sc. II (Paper I)

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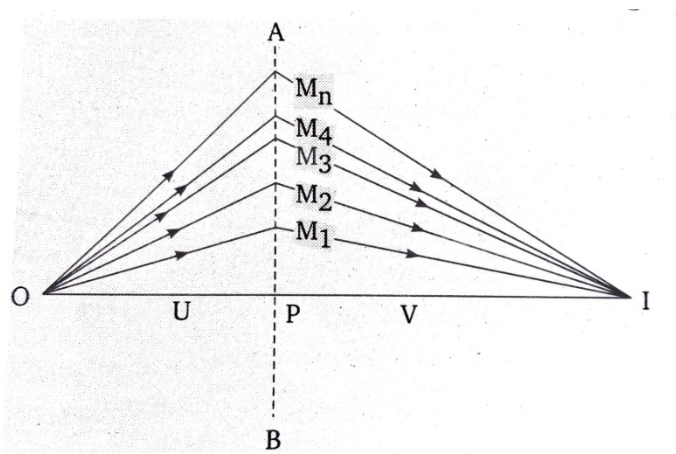
Zone plate

It is a transparent plate on which circles whose radii, proportional to the square roots of natural numbers 1, 2, 3… are drawn. The alternate annular zones thus formed are blocked. Such a plate behaves like a convex lens and produces an image of a source of light on the screen placed at a suitable distance.

Theory

Let 'O' be a luminous point object, emitting spherical waves of a wavelength λ whose effect at the point 'I' on the screen is required. Consider an imaginary plane through P of a transparent medium lying perpendicular to the paper and the line joining OI. Divide this plane in to zones bounded by circles having centers at P and to zones bounded by circles having centers at P and radii $PM_1=r_1$, $PM_2=r_2$, such that.



$$OM_1+IM_1=OP+IP+\frac{\lambda}{2}$$

$$OM_2+IM_2=OP+IP+2\frac{\lambda}{2}$$

$$OM_n+IM_n=OP+IP+n\frac{\lambda}{2}$$

The angular rings thus formed are half-period zones for the images I because length of the path of light through the corresponding points of any two consecutive zones differs by $\lambda/2$. To find the radius r_n of the n^{th} circle, we have,

$$OM_n+IM_n=OP+IP+n\frac{\lambda}{2}$$

Let, OP=u & IP=v

Now,

$$OM_n = (OP^2+PM_n^2)^{1/2}$$
$$= (u^2+r_n^2)^{1/2}$$
$$= u\,(1+ (r_n^2/u^2)^{1/2})$$
$$=u+ (r_n^2/2u)$$

because u is very large as compared to r_n and higher powers of $(r_n^2/2u)$ can be neglected.

Similarly, $IM_n= (v^2+r_n^2)^{1/2}=v+ (r_n^2/2v)$

Substituting these values in above equation, we have,

$$u+ (r_n^2/2u) +v + (r_n^2/2u) = u + v + n\frac{\lambda}{2}$$

$$r_n^2 \left(\frac{1}{u} + \frac{1}{v} \right) = n\,\lambda$$

$$\left(\frac{1}{u} + \frac{1}{v} \right) = n\,\lambda / (r_n^2)$$

Applying the sign convention, we have,

$$\left(\frac{1}{v} - \frac{1}{u} \right) = n\,\lambda / (r_n^2)$$

Or

$$r_n^2 = \frac{n\lambda\ uv}{u-v}$$

Since u, v & λ are constants.

$$\mathbf{r_n = \sqrt{n}}$$

This relation shows that the radii of half-period zones are proportional to the square root of natural numbers.

The area of the n^{th} zone is given by,

$$\boldsymbol{\Pi\,(r_n^2 - r_{n-1}^2) = \Pi\,\left\{\frac{n\lambda uv}{u-v} - \frac{(n-1)\lambda\ uv}{u-v}\right\} = \frac{\Pi\lambda\ uv}{u-v}}$$

How zone plate acts like a convex lens

If now the alternate zones, say even, are blocked, then the resultant displacement at I will become,

$A = A_1 + A_3 + A_5 + \text{-----}$

Which is many times greater than that due to all the zones and hence the intensity at I is very much increased. This explains the focusing action of the zone plate under these conditions I is said to be the image of the objects.

Focal length

The relation between u & v, the respective distance of the object and the image, is given by relation or

$$\left(\frac{1}{v} - \frac{1}{u} \right) = n\lambda \,/\, (r_n^2)$$

This is a result similar to the one found for a convex lens, i.e.,

$$\left(\frac{1}{v} - \frac{1}{u} \right) = \frac{1}{f}$$

Thus, the focal length of the zone plate**, $f = r_n^2/n\lambda$**

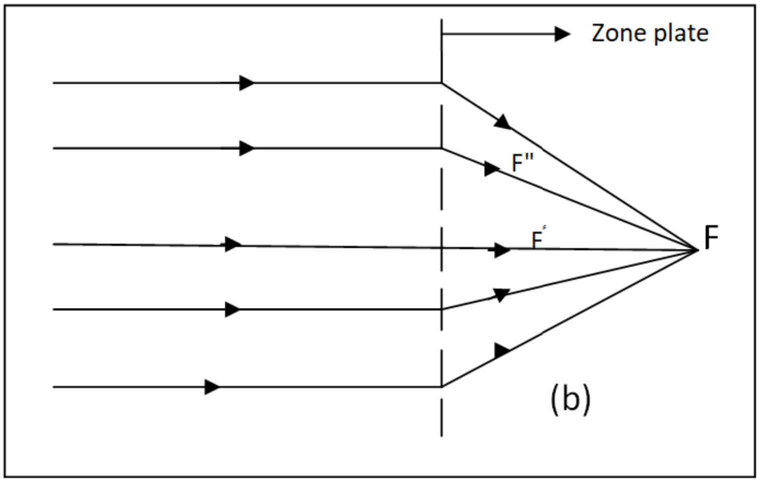
If the radius of the first half-period zone is 0.316mm & the wavelength $\lambda = 5*10^{-5}$, then the zone plate will behave like a lens of focal length 20 cm.

Zone plate has multiple foci

The focal length of a zone plate is given by the relation **$f = r_n^2/n\lambda$**,

Focal length of a zone plate is the distance measured along its axis at which the maximum is produced when the source of light is located at infinity i.e., when a plane wave front is incident on the zone plate. In actual practice between brightest maximum and the zone plate there are other points at distance f/3, f/5, f/7, etc. at which fainter maxima are produced. Thus unlike a converging lens, which has only one focus, a zone plate has multiple foci.

Let F be the focus of the zone plate where the brightest maximum is produced. If we move from F towards the zone plate, we get a point F' such that three zones can be constructed in each of the half-period zones of the given zone plate i.e., with respect to F' the whole of the zone plate can be divided in to 3n half-period zones.



As the first half-period zone of the original zone plate is divided in to three sub-zones contributing displacements (or amplitude) A_{11}, A_{12}, A_{13}, respectively, their net contribution to displacement by the first half-period zone (of the original zone plate).

$$A_1' = A_{11} - A_{12} + A_{13} = A_{13}$$

Similarly, net contribution to displacement by the second half-period zone.

$$A_2' = A_{21} - A_{22} + A_{23} = A_{23}$$

The contribution due to different zones of the original zone plate will again superimpose at F' to produce a maximum. This maximum will be less bright than that at F. since the total number of zones in to which the plate is now divided is 3n, therefore,

$$F' = (r_n^2/3n\ \lambda) = (f/3)$$

Moving further towards the zone plate we can have another point F" such that now each half-period zone of the original plate can be divided in to 5 sub-zones, As explained above, F" will also be a maximum but will be of a brightness even less than that of F'.

$$\text{Also, } f'' = (r_n^2/3n) = f/3$$

Thus, zone plate has multiple foci given by,

$$\mathbf{f = (r_n^2/n\ \lambda)\ ,\quad f'' = (r_n^2/3n\ \lambda)\ ,\ f''' = (r_n^2/5n\ \lambda)\ \&\ so\ on......}$$

Construction of zone plate

Zone plate is a system of areas corresponding to the half-period zones. To construct a zone plate, concentric circles whose radii are proportional to the square roots of the natural numbers are drawn on a sheet of white paper. The alternate zones are painted black and a very much reduced photograph of the drawing is obtained on a glass plate. The negative, when held in the light path from a distant point source, produces a large intensity at a point on its axis at a distance corresponding to the size of the zone and the wavelength of light used.

Positive and negative zone plates:

If odd zones are transparent and even zones are opaque, as shown in the figure (a). It is said to be a positive zone plate (Central zone is transparent).

If the even zones are transparent and odd zones are opaque, as shown in the fig (b). It is said to be negative zone plate (Central zone is opaque).

(a)
Positive Zone Plate

(b)
Negative Zone Plate

Zone plate has multiple foci

For a fixed distance of the object, a lens produces only one image, where as a zone plate produces a number of images. In other words, it has multiple foci. It is because the number of half-period zones contained in an area depends upon the position of the screen. If therefore, a point I_1 lies near to P than I so that the transparent rings contain three half-period zones, then the point I_1 will have again maximum illumination and thus will be the image of O. Similar will be the case when 5,7,---- half period zone are contained in transparent ring . The position of the various foci given by

$$f1 = (rn2/n\lambda)\ , \quad f2 = (rn2/3n\lambda)\ , f3 = (rn2/5n\lambda)\ \&\ so\ on \ldots\ldots$$

$$The\ intensity\ of\ the\ image, however, decreases\ as\ the\ focal\ length\ decreases.$$

Comparison between a zone plate and convex lens

Similarities:

- Like a lens, a zone plate forms an image of a point object placed on its axis and the distances of object and image are connected together by similar formulae in both the cases.
- Focal length of both varies with the wavelength λ and hence both show chromatic aberration.

Differences:

- The rays are brought to focus by refraction in case of a lens while the image is formed by diffraction in case of a zone plate.
- In the case of convex lens, all the refracted rays reach the focus simultaneously after traversing the same optical path, while in case of a zone plate, the diffracted rays take different times to reach the focus from different points on the plate and the rays from two successive transparent zones differ in path by λ.
- The image due to convex lens is more intense than that due to a zone plate.
- As the focal length of a zone plate is inversely proportional to λ, the violet rays come to focus at a larger distance from the zone plate than the red rays; while reverse is the case for convex lens where red rays come to focus at a longer distance.
- A convex lens has only one focus while a zone plate has a number of foci.

Phase reversal zone plates:

If the alternate zones of a zone plate; instead of being blackened, are covered by some transparent substance which introduces an additional optical path of λ 12, then the amplitude from successive zones will help each other and the intensity of the image approaches that for an equivalent lens. R.W. Wood, in 1898, prepared such a zone plate by coating a thin layer of gelatin solution and then immersing in a weak solution of potassium dichromate after allowing it to dry. Such zone plates are called phase reversal zone plates.

References:

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4. Physical Optics and Lasers by Tripathi and Singh