

FRESNEL'S DIFFRACTION

B.Sc. II (Paper I)

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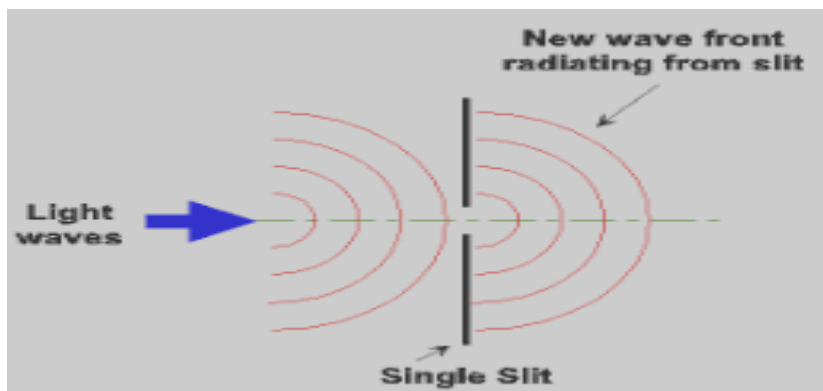
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DIFFRACTION OF LIGHT

Diffraction refers to various phenomena that occur when a wave encounters an obstacle or a slit. It is defined as the bending of waves around the corners of an obstacle or through an aperture into the region of geometrical shadow of the obstacle/aperture. The diffracting object or aperture effectively becomes a secondary source of the propagating wave.

The diffraction phenomenon is described by the Huygens–Fresnel principle that treats each point in a propagating wave-front as a collection of individual spherical wavelets. The characteristic bending pattern is most pronounced when a wave from a coherent source (such as a laser) encounters a slit/aperture that is comparable in size to its wavelength, as shown in the inserted image.



Difference between interference and Diffraction

Interference	Diffraction
1: <u>Interference</u> is due to the interaction of light coming from two different wavefronts originating from the same source.	1: <u>Diffraction</u> is due to the interaction of light coming from different parts of the same wavefront.

2: Interference fringes are of the same width.	2: Diffraction fringes are not of the same width.
3: All bright fringes are of the same intensity.	3: All bright fringes are not of the same intensity.
4: All point of minimum intensity are perfectly dark.	4: All point of minimum intensity is not perfectly dark.
5: The spacing between fringes is uniform.	5: The spacing between fringes is not uniform.

The diffraction phenomena in light are usually of two types

- Fresnel's Diffraction:
- Fraunhofer Diffraction:

If the source of light and screen are at finite distance from the obstacle, then the diffraction called Fresnel diffraction [Fig.(a)].

If the source of light and screen are at infinite distance from obstacle then the diffraction is called Fraunhofer diffraction [Fig.(b)]. The corresponding rays are not parallel.

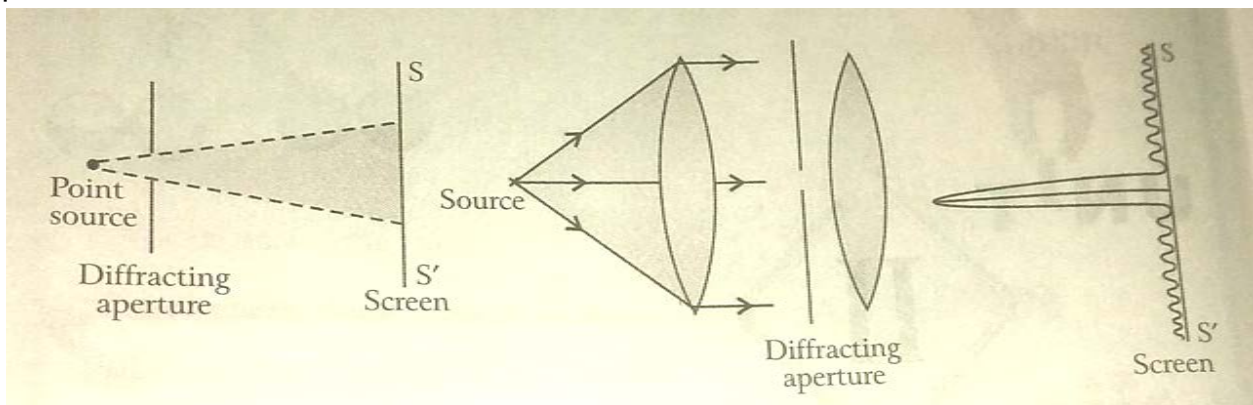


Fig.(a)

Fig.(b)

Fraunhofer Diffraction

Planar wave fronts

Observation distance is infinite. In practice, often at focal point of lens.

Fixed in position

Fraunhofer diffraction patterns on spherical surfaces.

Shape and intensity of a Fraunhofer diffraction pattern stay constant.

Fresnel Diffraction

Cylindrical wave fronts

Source of screen at finite distance from the obstacle.

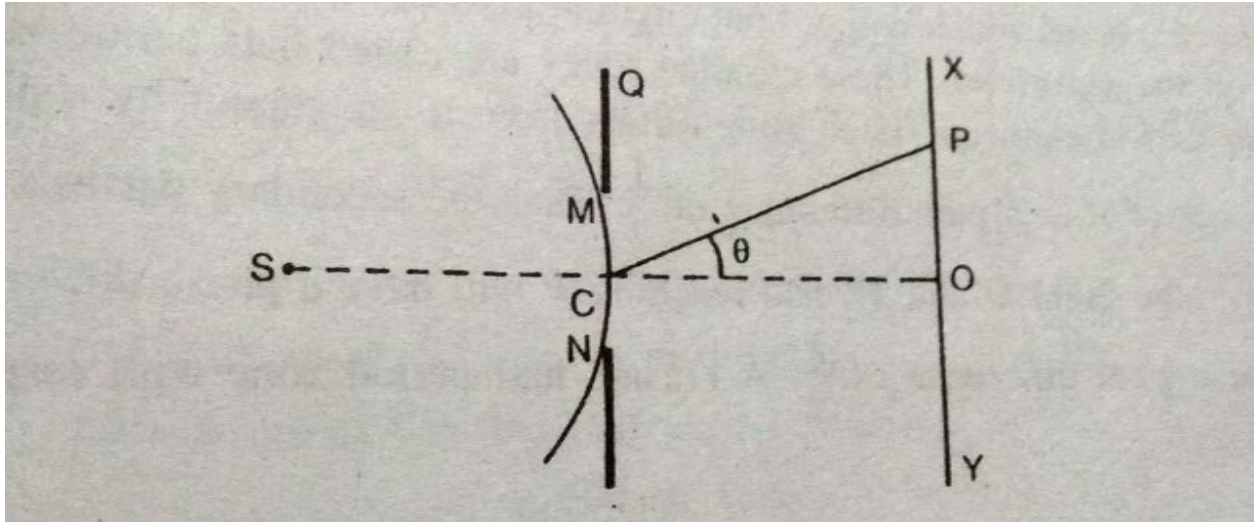
Move in a way that directly corresponds with any shift in the object.

Fresnel diffraction patterns on flat surfaces.

Change as we propagate them further 'downstream' of the source of scattering.

FRESNEL'S HALF PERIOD ZONES

According to Fresnel's the entire wave front can be divided into a large number of parts of zones which are known as Fresnel's half period zones. The resultant effect at any point on screen is due to the combined effect of all the secondary waves from the various zones

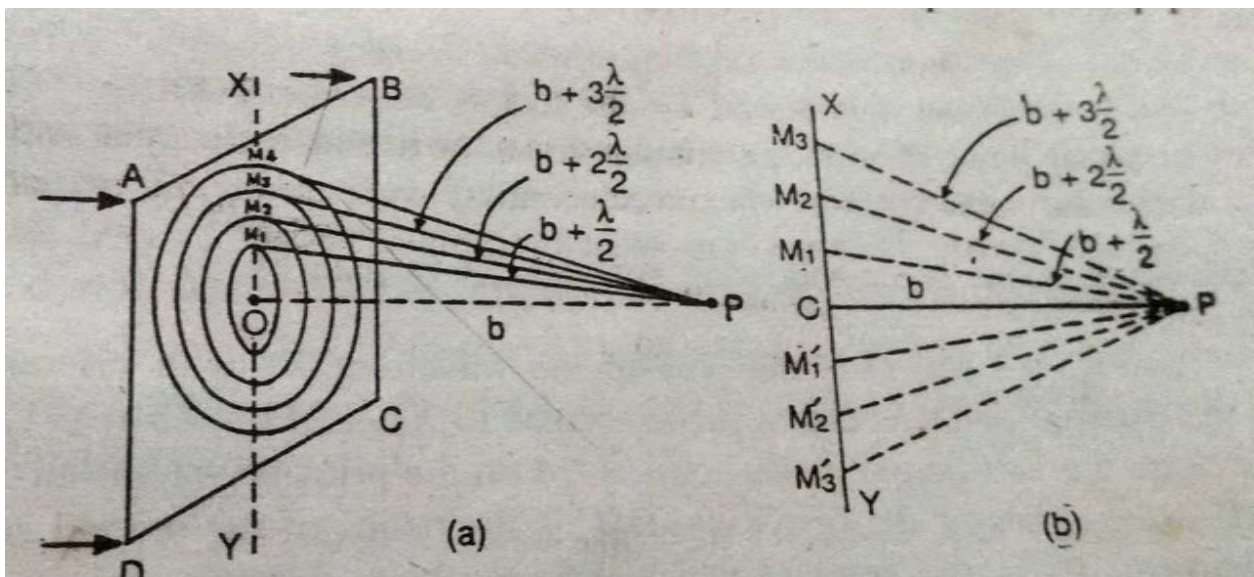


In figure, S is a point source of monochromatic light and MN is the small aperture. XY is the screen and SO is perpendicular to XY. MCN is the incident spherical wavefront due to the point source S.

Fresnel's assumed that-

1. A wavefront can be divided into a large no. of strips or zones. These zones known as Fresnel's half period zone.
2. Produce a resultant intensity of light at any point P. In order calculate the resultant intensity at a point due to wavefront.

Construction of half period zones



Let a source S emits a plane wave front ABCD travelling from left to right and has wavelength λ . Now, we wish to see the effect of wavefront point P at a distance b from the wavefront. Let us now divide the wavefront into Fresnel's zones with P as a centre and radii equal to $b + n \lambda/2$ ($n = 1, 2, 3 \dots$). It will cut areas of radii OM_1, OM_2, OM_3, \dots etc in fig.(a). The area enclosed between O and M_1 and M_2 .

Now draw concentric spheres on the wavefront as shown in Figure (a). The area between two spheres is called zone. The secondary waves from any two consecutive zones reach the point P with a path difference of $\lambda/2$ ($= T/2$) that is why the name half period zones. Here T stands for period. The point O is called the pole of the wavefront with respect to point P.

Radii of Half Period Zone: It will be

$$OM_1 = \sqrt{(b + \lambda/2)^2 - b^2} = \sqrt{b\lambda} \quad (\text{As } b \gg \lambda, \text{ so } \lambda^2 \text{ is neglected})$$

$$OM_2 = \sqrt{(b + 2\lambda/2)^2 - b^2} = \sqrt{2b\lambda}$$

Similarly,

$$OM_n = \sqrt{(b + n\lambda/2)^2 - b^2} = \sqrt{nb\lambda}$$

Thus radii are proportional to the square roots of natural numbers.

Area of Half Period Zone: The area of n^{th} zone will be

$$= \pi \{(b + n\lambda/2)^2 - b^2\} - \pi \{(b + (n-1)\lambda/2)^2 - b^2\}$$

$$= \pi \{b\lambda + \lambda^2(2n-1)/4\} = \pi b\lambda \quad \dots\dots(1)$$

$$(b \gg \lambda \text{ so } \lambda^2 \text{ is neglected})$$

The area of nth zone will be = $\pi b\lambda$

which says that area of each half period zone is nearly the same.

The distance of point P from half period zone:

It is

$$= [(b + n\lambda/2) + (b + (n-1)\lambda/2)] / 2 = b + (2n-1)\lambda/4 \quad \dots\dots(2)$$

Amplitude at point P due to one zone: It is given as

$$A_n = [\text{area of the zone} / \text{distance of point P from zone}] * \text{obliquity factor}$$

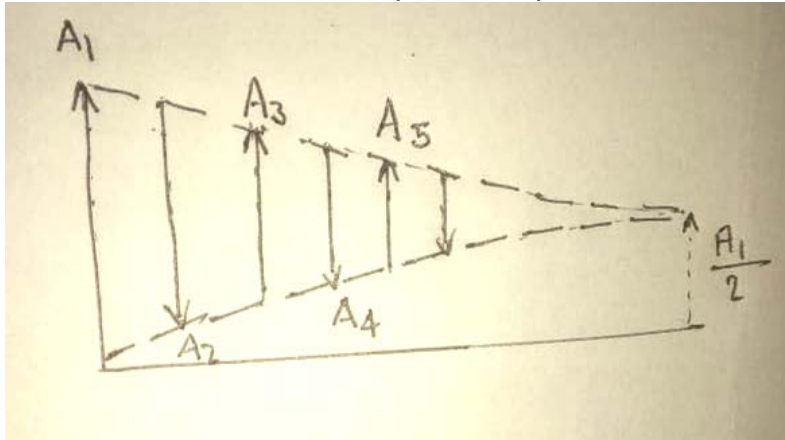
By using equation (1) and (2), we get

$$A_n = \pi \lambda (1 + \cos \theta_n)$$

Where $(1 + \cos \theta_n)$ is obliquity factor and θ_n is the angle between normal to the zone and line joining the zone to P. If n increases, $\cos \theta_n$ decreases and hence A_n also decreases.

Resultant amplitude of point P due to whole wavefront

As the path difference between the two consecutive zones is $\lambda/2$ so they are reaching in opposite phase. If A_1, A_2, A_3 and so on are the amplitudes at point P from various zones then the resultant amplitude at point will be



$$A = A_1 - A_2 + A_3 - A_4 \dots (-1)^{n-1} A_n$$

We can have

$$A_2 = A_1 + A_3/2 \quad \text{and} \quad A_4 = A_3 + A_5/2 \quad \text{and so on}$$

$$A = A_1/2 + A_n/2, \text{ for } n \text{ to be odd}$$

$$= A_1/2 + A_n/2 - A_n \text{ for } n \text{ to be even}$$

taking

$$A_{n-1} = A_n \quad \text{as } n \text{ is very large, then}$$

$$A = A_1/2 + A_n/2$$

$$A = A_1/2$$

Thus the amplitude due to a large wavefront at a point is just half that due to first half period Fresnel zone. The intensity will be

$$I = A^2$$

So,

$$I = A_1^2/4$$

thus, the intensity due to whole wavefront at O is only one fourth that due to the first half period zone along.

Intensity At An External Point Due To Cylindrical Wavefront

Consider a long and narrow slit SS' help parallel to plane of the paper. Let it be illuminated by a monochromatic source of light of wavelength λ (fig.a). The wavefront emanating from the slit will be cylindrical with the slit as the axis of the cylinder. O is the point at which the effect of the wavefront is to be find.

To determine the effect of this wavefront at O, let it be divided into elementary half period strips with respect to the point O. let XY represent a transverse cross section of the wavefront. Draw a normal OP on the surface of the cylindrical wavefront in fig.(b) then P is the pole of the screen. Let $SP=a$, $PO=b$. with O as centre and $(OP+ \lambda /2)$, $(OP+2 \lambda /2)$, $(OP+3 \lambda /2)$ as radii describe the arcs cutting the wave surface in M_1 , M_2 , M_3 , ...etc and N_1 , N_2 , N_3 etc, respectively. The lines through M_1 , N_1 , M_2 , N_2 etc., are draw parallel to the slit. This given us the strips of width PM_1 , M_1M_2 , M_2M_3 , and PN_1 , N_1N_2 , N_2N_3 Each parallel to the Slits. These strips are called first, second, third....., half period strips respectively. When viewed from point O they appear as shown in fig.(b)

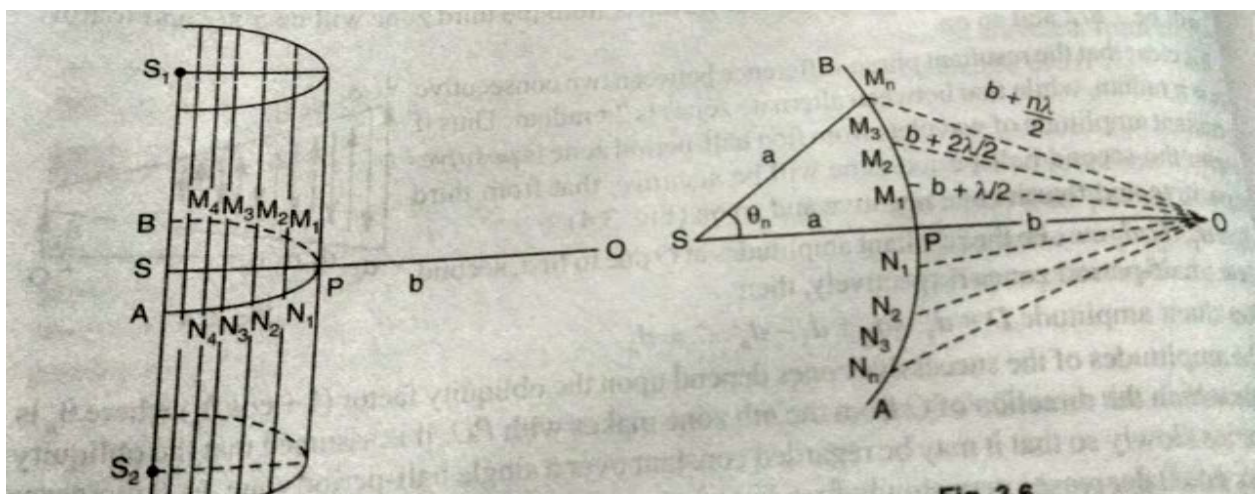


Fig.(a)

fig. (b)

Let $A_1, A_2, A_3 \dots$ etc., be numerical value of the amplitude at O due to first, second, third ... etc., half period strips on either side of the pole. The resultant amplitude at O due to either half of the wavefront will be.

$$A = A_1 - A_2 + A_3 - A_4 + A_5 \dots (-1)^n A_n$$

The alternate terms are with opposite signs because the waves from the consecutive strips differ in path by $\lambda/2$ and hence reach at O in the opposite phase. The series can be written as.

$$A = \frac{A_1}{2} + \left(\frac{A_1}{2} - A_2 + \frac{A_3}{2} \right) + \left(\frac{A_3}{2} - A_4 + \frac{A_5}{2} \right) + \dots$$

$$= A_1/2$$

(Since the expressions within the bracket reduce to zero)

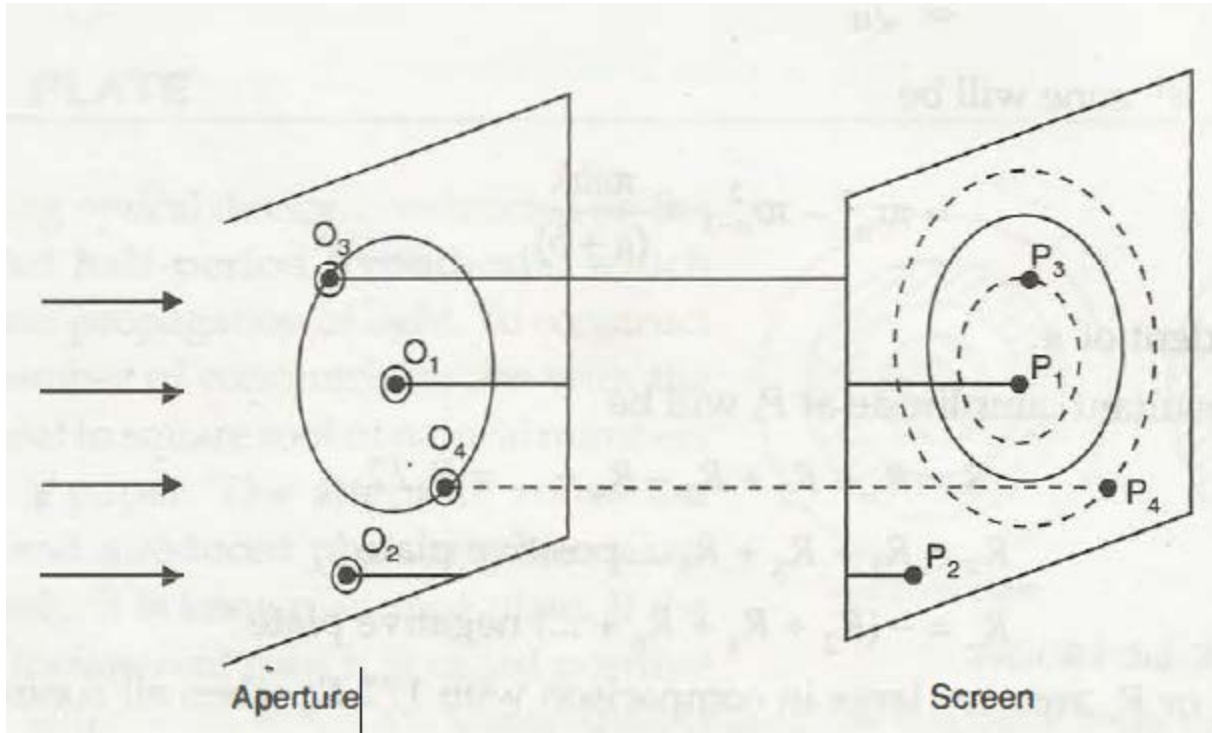
Thus, the resultant amplitude due to the entire cylindrical wavefront.

$$A = A_1/2 + A_1/2 = A_1$$

And the resultant intensity I at O due to entire cylindrical will be $(A_1)^2$

Rectilinear Propagation Of Light

Consider a circular or aperture on which a plane wavefront is falling normally as shown in the Figure. Let the screen is placed at same distance and parallel to the aperture. Let there are four points on the screen such as P_1 inside, P_2 outside, P_3 at boundary from inside and P_4 at the boundary of circle from outside. These points have corresponding the poles as O_1, O_2, O_3 and O_4 on the aperture. In case of O_1 sufficient number of zones are exposed so we get resultant amplitude $1/2 R_1$ at P_1 and so uniform illumination. For O_2 almost all the zones are blocked so we get resultant amplitude zero at P_2 and so complete darkness. For O_3 and O_4 few zones are exposed partially so resultant intensities at P_3 and P_4 is not uniform but decreases as we go from inside to outside.



FIGUR: . Rectilinear propagation of light

Thus, there is uniform illumination inside the circle, complete darkness outside but in between two dotted circle the intensity is not uniform. Hence one can conclude that rectilinear propagation of light is explained on the basis of wave theory.

References:

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2. Optics by Ajay Ghatak
3. Physical Optics and Lasers by J.P. Agrawal
4. Physical Optics and Lasers by Tripathi and Singh