DIFFRACTION GRATING

B.Sc. II (Paper I)

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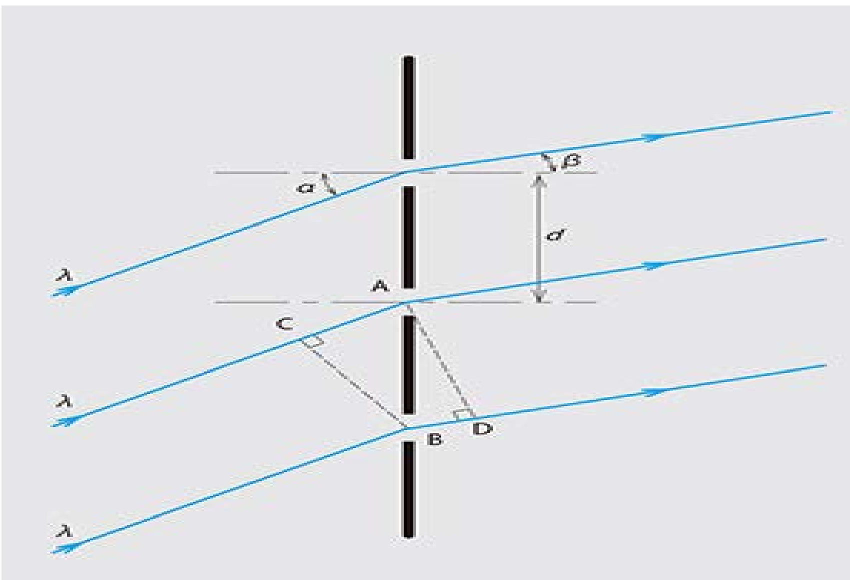
DIFFRACTION GRATINGS

A diffraction grating is an optical element that divides **(**disperses) light composed of lots of different wavelengths (e.g., white light) into light components by wavelength. The simplest type of grating is one with a large number of evenly spaced parallel slits. When white light enters the grating, the light components are diffracted at angles that are determined by the respective wavelengths (diffraction**)**. Picking out diffracted (reinforced) light makes it possible to select the required light component. In short, for parallel beams that enter neighboring slits.



Diffraction At N- Parallel Slits And Intensity Distribution

Let AB be the section of a plane transmission grating placed perpendicular plane of the paper (in below fig.(1)). Let e be the width of each transparent space and d the width of each opaque portion. Then (e+ d) is the grating element. Two points separated by the distance (e +d) in two consecutive slits are known as corresponding points. If the spacing between the lines is of the order of the wavelength of light, then an appreciable diffraction of light is produced.

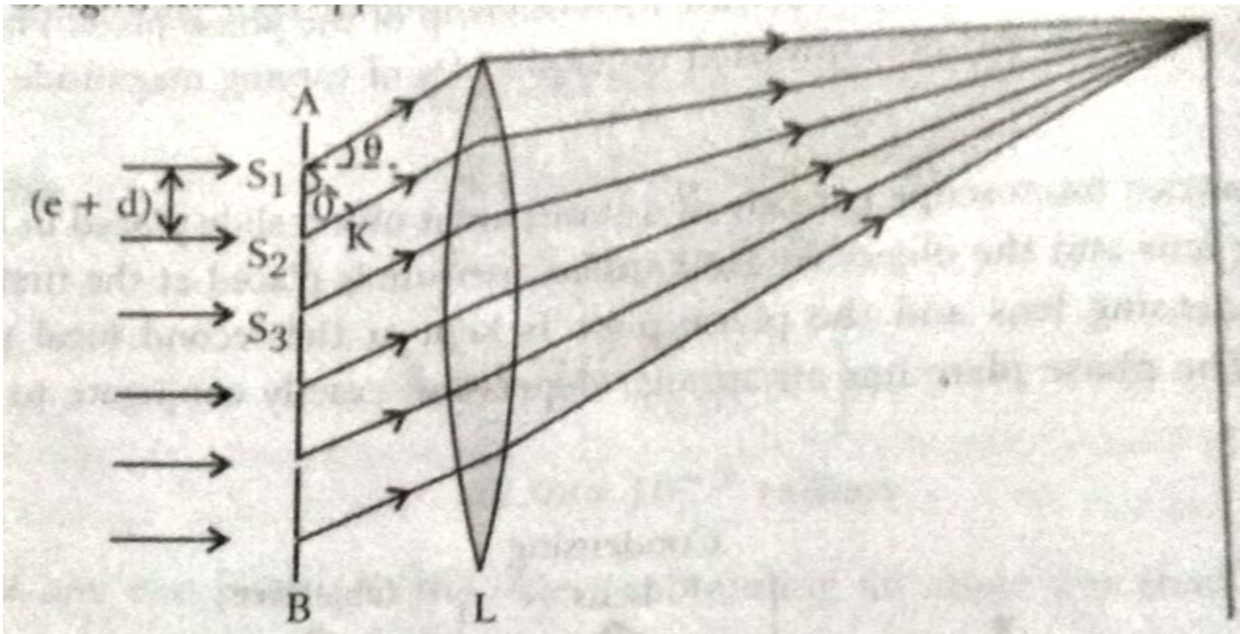


Fig.(1)

Let us parallel beam of monochromatic light of wavelength λ be incident normally on the N slits. By Huygen's principle, all the point in each silts send out secondary wavelets in all directions. By the theory of Frounhofer diffraction at a single slit, the wavelets from all points in a slit diffraction in direction θ are equivalent to a single wave of amplitude $\frac{A_0}{\alpha}\sin\alpha$, starting from the middle points of the slit, where $\alpha = \frac{\pi\ e \sin\theta}{\lambda}$

Thus, if N be the total number of slits, the diffraction rays all the slits are equivalent to N parallel rays, one from the middle points S_1, S_2, S_3, ….. of each slit. Let S_1K be perpendicular to S_2K. Then path difference between the rays from the slit S_1 and S_2 is

$$\mathbf{S_2}K = S_1S_2 \sin\theta = (e+d)\sin\theta$$

And the corresponding phase difference= $\frac{2\pi}{\lambda}(e+d)\sin\theta$

Let this phase difference = $\frac{\pi}{\lambda}(e+d)\sin\theta = 0$ and the resultant of vibration from each slit is $A_0 \frac{\sin\theta}{\alpha}$

Where = $\alpha = \frac{\pi}{\lambda} e \sin\theta$. Extending the application of these results to N-slits. The problem reduce to finding the resultant amplitude of N vibration in a direction θ , each of amplitude $A_0 \frac{\sin\theta}{\alpha}$, but the phase increases in arithmetical progression, the common phase difference being $\frac{2\pi}{\lambda}(e+d)\sin\theta$. therefore by the process of vector addition, there resultant amplitude due to all slits in the direction θ is,

$$A = A_0 \frac{\sin\theta}{\alpha} * \frac{\sin[N\frac{\pi}{\lambda}(e+d)\sin\theta]}{\sin[\frac{\pi}{\lambda}(e+d)\sin\theta]}$$

$$= A_0 \frac{\sin\theta}{\alpha} * \frac{\sin N\beta}{\beta}$$

Hence, the intensity is given by, $I = A^2 = (A_0 \frac{\sin\alpha}{\alpha} . \frac{\sin N\beta}{\beta})^2$ …… (1)

The first factor of eq(1), $\left(A_0 \frac{\sin\alpha}{\alpha}\right)^2$ correspond to the intensity of diffraction pattern due to a single slit, while the second factor $\left(\frac{\sin N\beta}{\beta}\right)^2$ gives the distribution of intensity due to all the slits.

POSITION ON PRINCIPAL MAXIMA

Principal maxima are obtained for maximum value of second term $\left(\frac{\sin N\beta}{\beta}\right)^2$ for its maximum value

$$\sin\beta = 0$$

$$\beta = \pm n\pi$$ where n=1,2,3….

$$\frac{\sin N\beta}{\sin\beta} = \frac{0}{0}$$ i.e, indeterminate.

To find its value, we use the usual method of differentiating the number and the denominator.

$$\lim_{\beta\to\pm n\pi} \frac{\sin N\beta}{\sin\beta} = \lim_{\beta\to\pm n\pi} \frac{N\cos N\beta}{\cos\beta} = \frac{N\cos N(\pm n\pi)}{\cos(\pm n\pi)} = \pm N$$

Then the intensity,

$$I = \left(A_0 \frac{\sin\alpha}{\alpha}.\right)^2 N^2$$

So,the intensity is proportional to N and it is maximum. The maxima are very intense and are called principal maxima.

$$\beta = \pm n\pi$$

$$\frac{\pi}{\lambda}(e+d)\sin\theta = \pm n\pi$$

$$(e+d)\sin\theta = \pm n\lambda$$ **……. (2)**

$$where\ n = 0,1,2,3 \ldots$$

n =0, we get the zero order maximum.

n =$\pm1, \pm2, \pm3$ …. we get the first, second and third order maxima respectively.

Position of minima A series of minima occur when sin Nβ =0, provided that sin$\beta \neq 0$

$$\frac{\sin N\beta}{sin\beta} = 0$$

from eq(1), we have

$$I = (A_0 \frac{\sin \alpha}{\alpha} .)^2 .0 = 0$$

Which is a minimum. The position of minima correspond to

$$\sin N\beta = 0 = \pm \sin m\pi$$

$$N\beta = \pm m\pi$$

$$\frac{N\pi}{\boldsymbol{\lambda}} (e + d) \sin\theta = \pm m\pi$$

$$N (e + d) \sin\theta = \pm m\boldsymbol{\lambda} \qquad \textbf{..... (3)}$$

where m has all the integral values except 0, N, 2N,….. Nn, because for these values of β =0, which give principal maxima. It is clear from above that, m=0 gives a principal maximum, and m=1,2,3 …..(N-1) gives minima and then m=N given again a principal maxima. Thus, there are

(N-1) minima between consecutive principal maxima.

Position of secondary maxima- As there are N-1minima between two consecutive maxima, there must be N-2 other maxima between two principal maxima. These are called secondary maxima. These secondary maxima are less intense than the principal maxima. Their position are obtained by differentiating eq.(1) w.r.t β and equating to zero. Thus,

$$\frac{dI}{d\beta} = (A_0 \frac{\sin \alpha}{\alpha} .)^2 \cdot 2[\frac{\sin N \beta}{\sin \beta}] \frac{N \cos N\beta - \sin N\beta \cos \beta}{sin\, 2_{\beta}} = 0$$

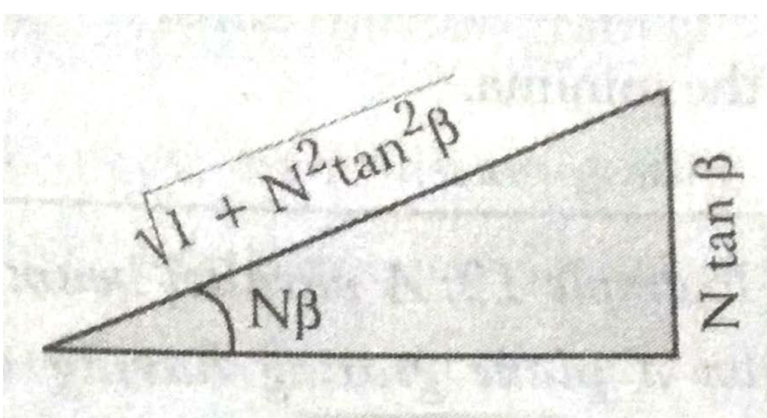


Fig.(2)

$$\text{N}\cos\beta - sinN\beta\ cos\beta = 0$$

$$\text{N cos N}\beta = sinN\beta\ cos\beta$$

$$n \cot N\beta = N \cot \beta$$

$$\tan N\beta = N \tan \beta$$

Hence, to find their intensity, the fraction $\frac{\sin\, 2_{N\beta}}{\sin\, 2_{\beta}}$ has to determined subjected to the condition

$$\tan N\beta = N \tan \beta$$

For this we use the triangle shown in fig.(2)

$$\sin N\beta = \frac{N \tan \beta}{\sqrt{1 + N^2 \tan 2_{\beta}}}$$

$$\frac{\sin\, 2_{N\beta}}{\sin\, 2_{\beta}} = \frac{N^2 \ \tan\, 2_{\beta}}{(1 + N^2 \ \tan\, 2_{\beta})\sin\, 2_{\beta}} = \frac{N^2}{\cos\, 2_{\beta + N^2} \ \sin\, 2_{\beta}}$$

$$= \frac{N^2}{1 + (N^2 - 1)\sin\, 2_{\beta}}$$

It show that the intensity of secondary maxima is proportional to $\frac{N^2}{1+(N^2-1)\sin\, 2_{\beta}}$, while that of principal maxima is proportional to N^2.

The intensity ratio of the secondary maxima and the principal maxima i.e.,

$$\frac{\textbf{intensity of secondary maxima}}{\textbf{intensity of principal maxima}} = \frac{\mathbf{1}}{\mathbf{1} + (\boldsymbol{N^2} - \mathbf{1})\ \mathbf{sin}\ \mathbf{2}_{\boldsymbol{\beta}}},$$

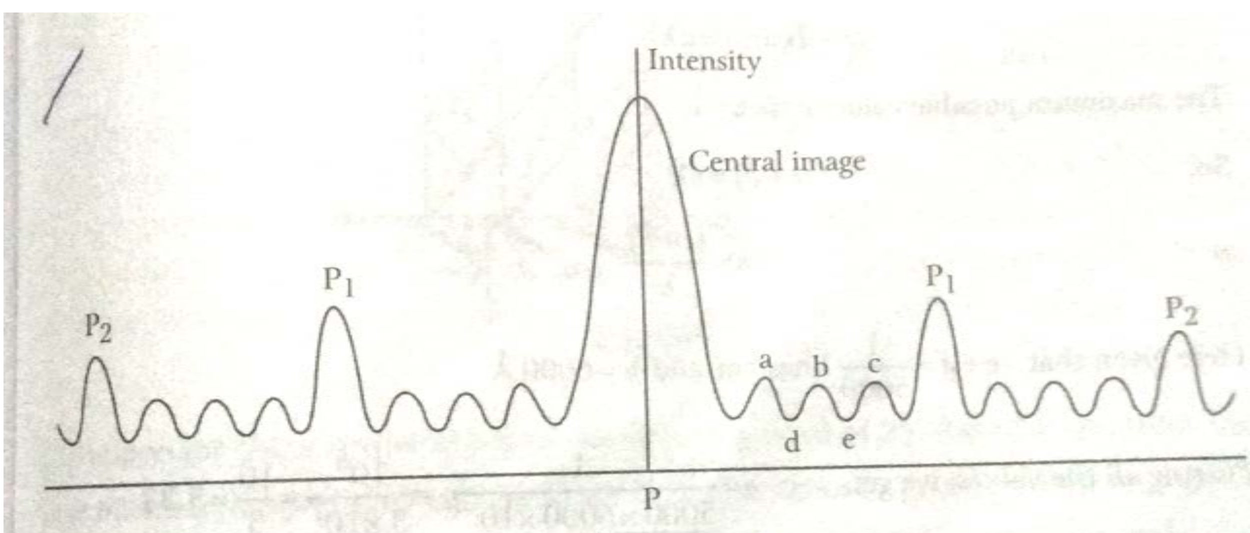


Fig.(3)

Plane diffraction grating

Consider an optically plane surface with its surface made reflecting by silvering or aluminizing. If a large number (N) of parallel straight lines are scratched on it. Each of width b and leaving equal widths a clear between them, we get a plan refection grating.

The reflection grating is use to obtain spectra just like the plane transmission grating, the theory and action being the same. Reflection grating has the particular advantage that it can be used also in the ultra-violet and infrared regions where most materials become opaque. Hence it is very widely used.

Blazed Grating

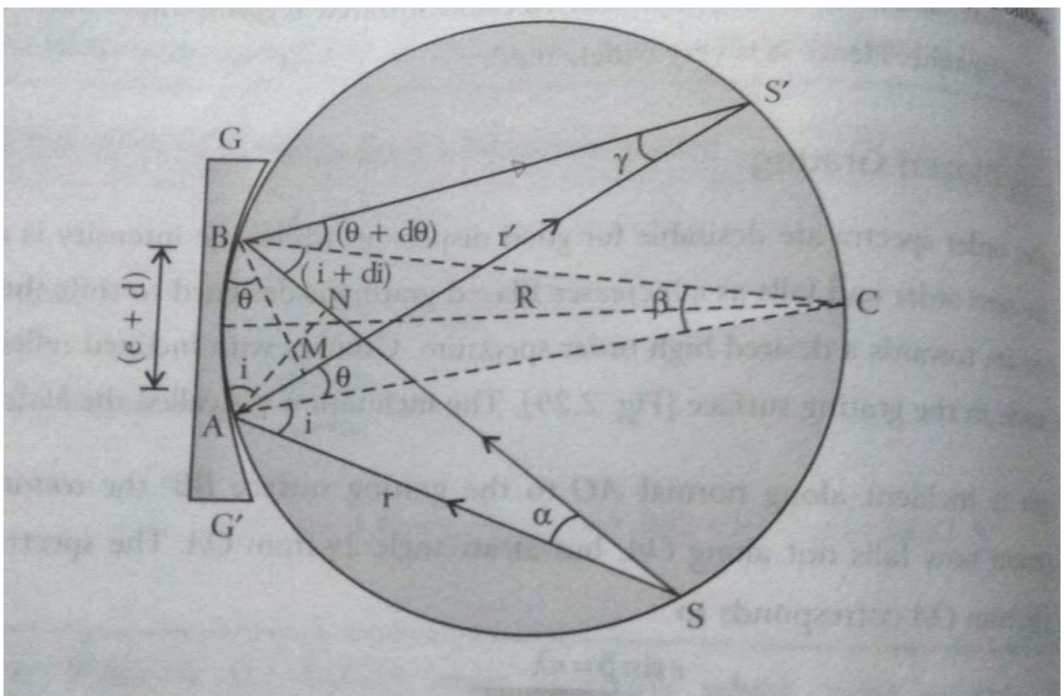
A blazed grating also called echelette grating is a special type of diffraction grating. It is optimized to achieve maximum grating efficiency in a given diffraction order. For this purpose, maximum optical power is concentrated in the desired diffraction order while the residual power in the other orders (particularly the zeroth) is minimized. Since this condition can only exactly be achieved for one wavelength, it is specified for which blaze wavelength the grating is optimized (or blazed). The direction in which maximum efficiency is achieved is called the blaze angle and is the third crucial characteristic of a blazed grating directly depending on blaze wavelength and diffraction order.

Concave Reflecting Grating

The wavelength of spectral line can be determined accurately with a plane transmission grating, knowing the grating constant, the diffraction angle and the order n. use of a plane transmitted grating requires two lenses, the collimating lens and the telescope objective. The collimating lens given a parallel beam of light incident on the grating surface and the telescope objective focuses the diffracted beam. The use of two lenses if they are not perfectly achromatic, make the spectrum more complex due to chromatic aberration present in the lenses. Rowland developed the concave reflecting grating, the use of which dispenses the use of both lenses. A concave grating consists of a polished spherical concave surface of a metal. Polished concave surface is ruled with equally spaced fine parallel line. When light falls on concave grating, then after diffraction, it is automatically focused without the use of lenses. In such a way, the effect of chromatic aberration is completely eliminated.

Theory of Concave Reflecting Grating

Let GG' be a concave grating having of curvature at C (in below fig.). Let A and B be two corresponding points of the grating so that AB =(e+ d) is the grating element. Let S be a narrow slit perpendicular to the plane of the paper and illuminated by monochromatic light of wavelength. Let SA and SB be two rays incident on the grating at angle i and i + di respectively. AS 'and BS' are the corresponding diffracted rays with angel of diffraction and respectively.



=(SB+BS')-(SA+AS')

=(SB-SA)-(AS'-BS')

=BN-AM

=AB(sin i-AB sin *θ)* *[from above fig.]*

= (e+d)(sini-sin *θ)*

[AB =(e+d)]

For nth order spectrum, a maximum at S' occurs only when

$$(e+d)(\sin i - \sin \theta) = n \qquad \ldots\ldots. (1)$$

In order to from a diffraction image at S', the path defference between any two consecutive rays should be constant i.e.,

$$=(e+d)(\sin i - \sin \theta) \quad = \text{constant}$$

On differentiating eq.(1), we get

$$(\cos i\, di - \sin \theta\, d) = 0 \qquad \ldots\ldots. (2)$$

But, from above fig. we have

$$\alpha + i = \beta + i + di \qquad \therefore di = \alpha - \beta$$

$$\beta + \theta = \gamma + \theta + d\theta \qquad \therefore d\theta = \beta - \gamma$$

Pitting these values in eq. (2), we get

$$\cos i(\alpha - \beta) - \cos\theta(\beta - \gamma) = 0 \qquad \ldots. (3)$$

Let SA=r, AS'=r' and radius of curvature of the grating be R, then

$$r.\alpha = AN = (e+d)\ \cos i$$

$$R\beta = AB = (e+d)$$

$$r'\gamma = BM = (e+d)\cos\theta$$

$$\therefore \qquad \alpha = (e+d)\cos i, \quad \beta = \frac{e+d}{R}\ , \qquad and\ \gamma = \frac{(e+d)\cos\theta}{r'}$$

Putting these values in eq. (3), we get

$$\cos i\left[\frac{\cos i}{r} - \frac{1}{R}\right] - \cos\theta\left[\frac{1}{R} - \frac{\cos\theta}{r'}\right] = 0 \qquad \ldots\ldots. (4)$$

It is the general eq. for the position of S' which shows that,

If, r=R cos i than r'=R $\cos\theta$

i.e., If S lies at the circumference of a circle of radius R, then S' also lies on the same circle. Thus, we can say that if the slit and the concave grating are placed at the circumference of a circle whose diameter is equal to the radius of curvatures of the grating, then the spectra are focused on the circumference of the same circle.

Different methods of mounting of concave grating

To use a concave grating, the slit, grating and eyepiece can be placed in different position known as mounting. For concave grating, there are mainly three type of mounting.

1. Rowland Mounting

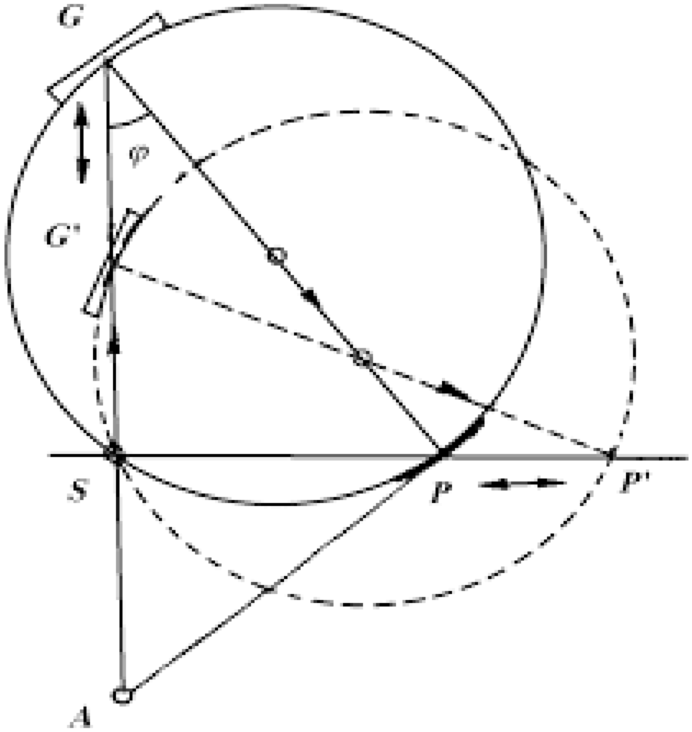
2. Paschen-Runge Mounting

3. Eagle Mounting

Rowland Mounting

The principle of Rowland Mounting **is** illustrated in below fig. G is the concave grating and P is the plate holder. The grating and the plate holder are mounted at the ends of a beam of length R equal to the radius of curvature of the grating surface. This beam GP can slide along two rails **SX** and SY. G, P and G, P' represent two positions of the beam. The slit S is set at the point of intersection of the rails SX and SY.

With an arrangement of this type, the region of the spectrum imaged at P can be altered by sliding the beam. Sliding the beam alters the angel of incidence i. the spectrum obtained with this arrangement is nearly normal because the angle is nearly zero. For any position of the beam the spectrum is imaged at P.



From the eq.

$$(a + b)(\sin i - \sin \theta) = n \boldsymbol{\lambda}$$

If

$$(a + b) \sin i = n \boldsymbol{\lambda}$$

Here (a + b) is a constant. For a given order

$$\sin i \propto \boldsymbol{\lambda}$$

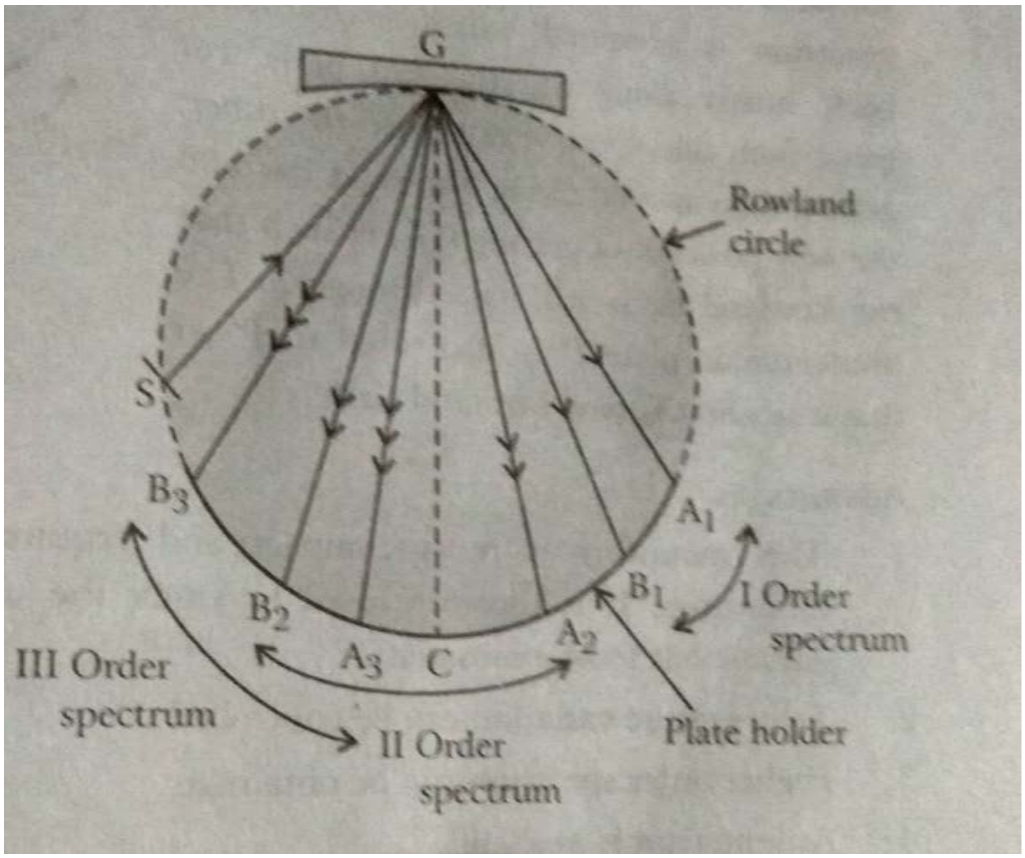
But $$\sin i \propto SP$$

$\therefore$ $$SP \propto \boldsymbol{\lambda}$$

Thus with a mounting of the type, which is mostly of historical interest, it is possible to calibrate the rail SP for the wavelength of spectral lines.

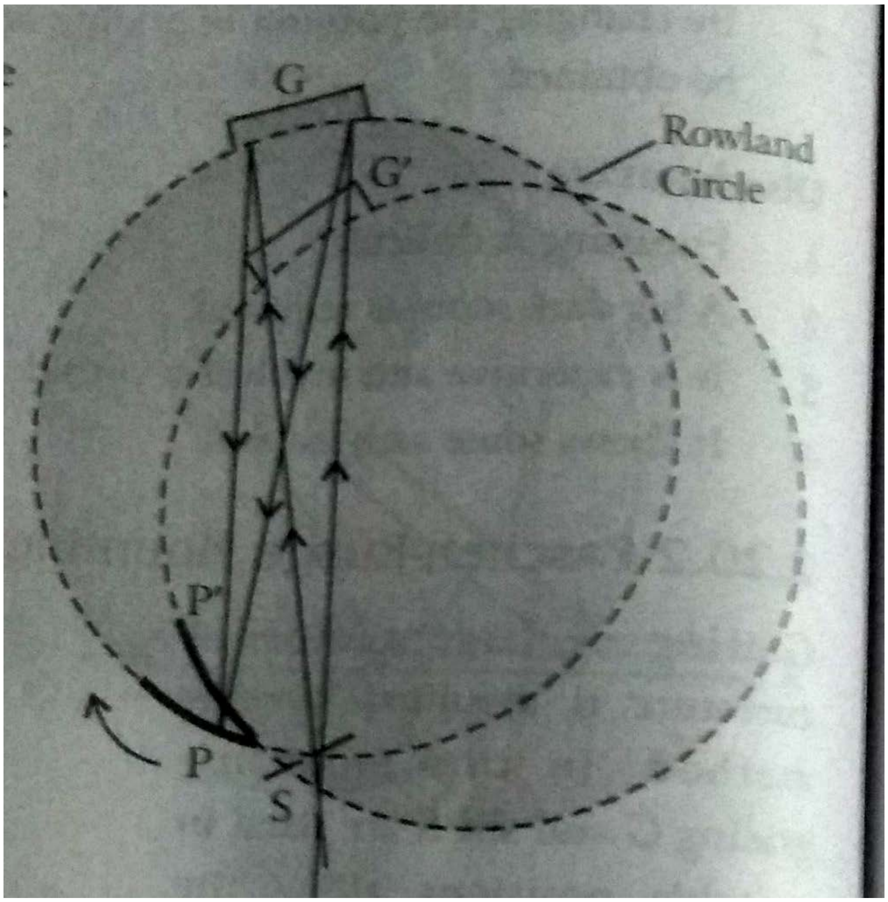
Paschen-Runge Mounting:

Grating of large radius of curvature is mounted by this method. In this mounted grating G and slit S are fixed in suitable positions along the Rowland circle so as to give a desired angle of incidence, the spectra of different orders are imaged on the circumference of the circle. In fig. A_1B_1 is the first order spectrum, A_2B_2 and A_3B_3 are the second order and third order spectrum respectively. So, with this mounting several orders of the spectrum can be photograph simultaneously. The photograph plate is held to a circular steel frame the Rowland circle.



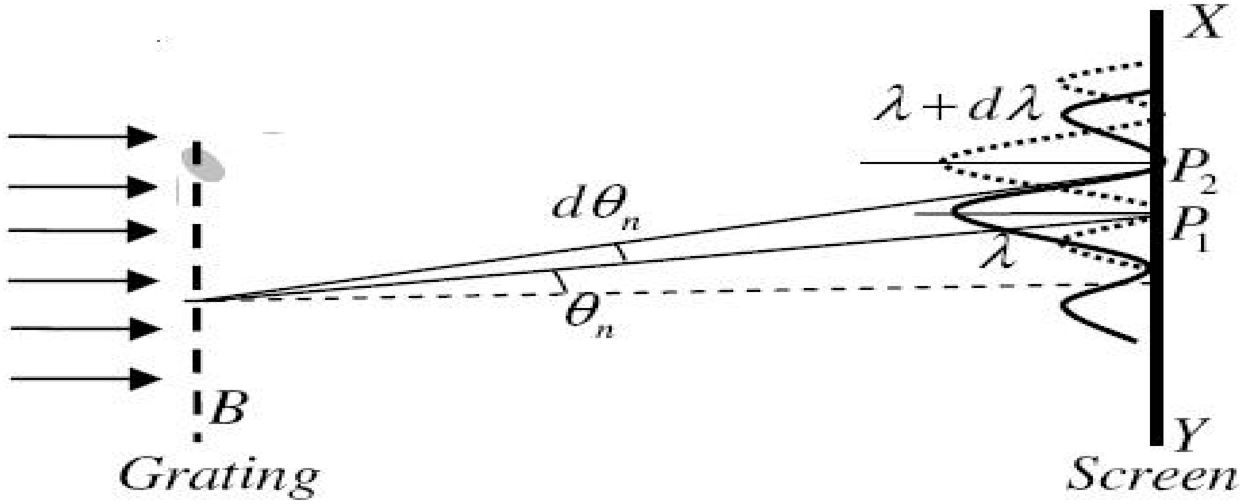
Eagle Mounting

In this mounting, the slit S and the photograph plate P are mounted very close together at one end of a rigid bar and on the other end of bar concave grating is mounted (in below fig.). In addition to translational motion the grating can be rotate also with the help of screw. All the three lie on the Rowland circle. Thus, only that part of the spectrum is obtained w**hich** is diffracted back, nearly along the incident path. For going from one spectral region to the other, grating G is moved along the rigid to bar and the new position of grating G' is such that the Rowland circle still pass through S. The photograph plate P is also tilted to P' so that it lies on the new Rowland circle.



Resolving power of a grating

It is defined as the capacity of a grating to from separate diffraction maxima of two wavelengths which are very close to each other. It is measured **λ / dλ** where d**λ** is the smallest difference in two wavelengths which are just resolvable by grating and is the wavelength of either of them or mean wavelength.



Let *AB* represent the surface of a plane transmission grating having grating element

(e+ d) and *N* total number of slits. Let a beam of light having two wavelengths λ and λ +d λ is normally incident on the grating. Let P_1 is *nth* primary maximum of a spectral line of wavelength λ at an angle of diffraction $\boldsymbol{\theta}$ and P_2 is the n^{th} primary maximum of wavelength

λ + dλ at diffracting angle $\boldsymbol{\theta} + \boldsymbol{d\theta}$ **.**

According to Rayleigh criterion, the two wavelengths will be resolved if the principal maximum **λ + dλ** of n^{th} order in a direction $\theta + d\theta$ falls over the first minimum of n^{th} order in the same direction $\theta + d\theta$. Let us consider the first minimum of λ of n^{th} order in the direction $\theta + d\theta$ as below.

The principal maximum of **λ** in the θ direction is given by

$$(e + d)\sin\theta = n\boldsymbol{\lambda} \qquad \text{....... (1)}$$

The equation of minima is N $(e + d)\sin\theta = m\boldsymbol{\lambda}$*,* where m takes all integers except 0, N, 2N, …, nN, because for these values of m, the condition for maxima is satisfied. Thus first minimum adjacent to n^{th} principal maximum in the direction $\theta + d\theta$ can be obtained by substituting the value of 'm' as (nN+1). Therefore, the first minimum in the direction of

$\theta + d\theta$ is given by

$$\text{N } (e + d)\sin\theta = (nN + 1)\boldsymbol{\lambda}$$

$$(e + d)\sin\theta = (n + \tfrac{1}{N})\boldsymbol{\lambda} \qquad \text{…. (2)}$$

The principal maximum of **λ+dλ** in direction $\theta + d\theta$ is given by

$$(e + d)\sin(\theta + d\theta) = n(\boldsymbol{\lambda} + \mathbf{d}\boldsymbol{\lambda}) \qquad \text{.... (3)}$$

Dividing eqn (2) by eqn (3), we get

$$\left(n + \frac{1}{N}\right)\boldsymbol{\lambda} = n(\boldsymbol{\lambda} + \mathbf{d}\boldsymbol{\lambda})$$

$$\frac{\boldsymbol{\lambda}}{\mathbf{d}\boldsymbol{\lambda}} = \mathbf{nN}$$

Thus, <u>the resolving power of a grating is equal to the product of the order of the spectrum and the total number of lines on the grating.</u>

Maximum resolving power = $\frac{\mathrm{N(a+b)}}{\boldsymbol{\lambda}} = \frac{\mathrm{W}}{\boldsymbol{\lambda}}$

Where, W = $\mathrm{N(a + b)}$ is the total width of ruled space in the grating. The resolving power of the grating would not be affected if the number of lines N in a given width of ruled space is changed.

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