STATISTICAL PHYSICS

(Part-2)

B.Sc. III (paper-1)

Unit-II

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PROBABILITY

The probability of an event may be defined as the ratio of the number of cases in which the event occurs to the total number of cases, i.e.

$$\textbf{Probability of an event} = \frac{\textbf{Number of cases in which the event occurs}}{\textbf{Total number of cases}}$$

Thus, if an event can happen in a ways and fails to happen in b ways, then the probability of happening of the event = $\frac{a}{a+b}$ and the probability of failing of the event = $\frac{b}{a+b}$

Here it has been assumed that (a+ b) ways have the same chance of occurrence.

Ex- let there are five boys A, B, C, D and E who want to get a particular white ball. Clearly, the probability of A getting the white ball is 1/5. If there is red ball which is to be distributed similarly; then the probability of A getting it again is 1/5. Thus the probability of A getting both ball simultaneously is 1/5*1/5=1/25 because the desired event takes place only once out of 25 equally likely ways in which the composite event take place. Therefore, we conclude that the probability of a composite event is equal to the product of the probabilities of the individual and independent events.

Q. **1) There are 6 pillows in a bed, 3 are red, 2 are yellow and 1 is blue. What is the probability of picking a yellow pillow?**

Ans -The probability is equal to the number of yellow pillows in the bed divided by the total number of pillows, i.e. 2/6 = 1/3.

Probability of an Event

Assume an event E can occur in r ways out of a sum of n probable or possible equally likely ways**.** Then the probability of happening of the event or its success is expressed as;

$$P(E) = r/n$$

The probability that the event will not occur or known as its failure is expressed as:

$$P(E') = n-r/n = 1-r/n$$

E' represents that the event will not occur.

Therefore, now we can say;

$$\mathbf{P(E) + P(E') = 1}$$

This means that the total of all the probabilities in any random test or experiment is equal to 1.

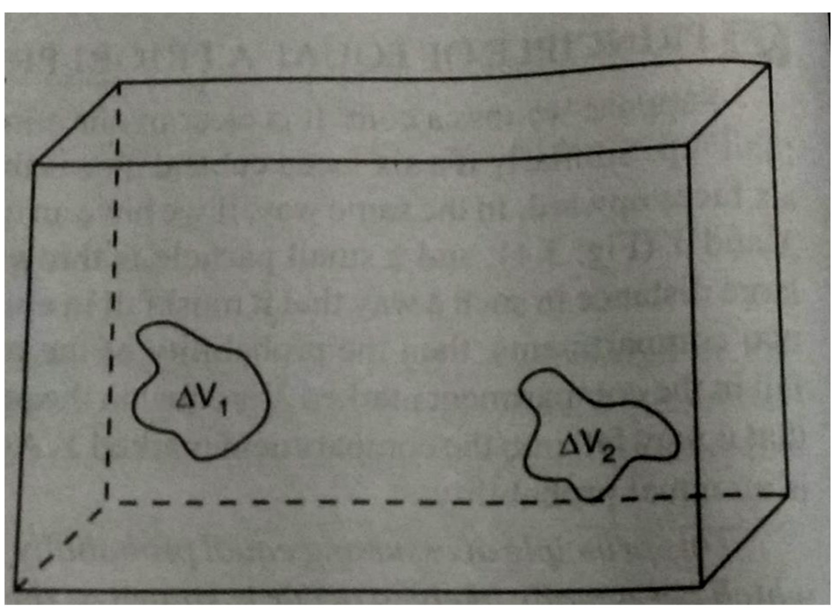
Note- the probability P of a ramdom event lies between 0 and 1 i.e.,

$$\mathbf{0 \leq \boldsymbol{P} \leq 1}$$

Some Basic Rules Of Probability Theory

1) Additive Law of Probability

This law is applicable to mutually exclusive events. Two or more events are said to be mutually exclusive if the occurrence of any one of them prevent the occurrence of others. Such events never occurs simultaneously.



For example, consider two small non overlapping regions ΔV_1 and ΔV_2 in a box of volume V as shown in fig. A particle in ΔV_1 rules out the possiblility of its being present, at the same instant in ΔV_2 and vice -versa. The events, thus, are mutually exclusive.

Suppose in N trails the particle is found m_1 times in ΔV_1 and m_2 times in ΔV_2. The probabilities of finding particles in the two regions are $p= m_1/n$ and $p=m_2/N$ respectively.

The number of times that the particle will be found at least one of the two regions in N trials is (m_1+m_2). Hence, the probability will be

$$P\ (\Delta V_1 \text{ or } \Delta V_2) = m_1+m_2/N$$

$$=\frac{m_1}{N} + \frac{m_2}{N}$$

$$=p_1+p_2$$

This law can be generalized to any number of mutually exclusive events, giving probability say ,p_1, p_2, p_3,……..p_N then the probability that any one of them occurs is the sum of the probability of these events, i.e**.,**

$$\mathbf{P=p_1+p_2+p_3+\ldots\ldots\ldots+p_N = \sum_{i=0}^{N} p_i}$$

This is known as additive law of probability.

2) **Multiplication Rule: Joint Probability**

In the calculations of probability, we sometimes come across random events; such that the probability of occurrence of one does not affect the probability of occurrence of the other. For example in above fig. the probability that a molecule A gets into ΔV_1, at a particular instant is p1=$\Delta V_1/V$. The probability of another molecule get into volume V_2 at the same instant is $p_2=\Delta V_2/V$, regardless of whether or not the molecule A gets into ΔV_1. We want to calculate the probability of joints occurrence of these events. Suppose in N trails, the molecule A is found m times in ΔV_1. If p_2 is the probability that B gets into ΔV_2, irrespective of the presence of A in ΔV_1, the no. of times the two events will occur simultaneously is m p_2. Thus, the joint probability of occurrence of these two events, is

$$\mathbf{P = mp_2/N}$$

$$\mathbf{= m * p_2/N}$$

$$\mathbf{P = p_1 * p_2} \qquad (p_1=m/N)$$

Thus, probability of joint occurrence of two independent events is equal to the product of the probabilities of each of these independent events.

Example- we throw a die twice and obtain two numbers. What is the probability that these numbers are 6 and 4 precisely in that order?

Solution. The probability that the first throw given a 6 is 1/6. Similarly, the probability that the second throw given a 4 is also 1/6. There two events are independent.

Required probability=1/6*1/6=1/36

3) Conditional probability

The probability for an event A to occur the condition that event B has occurred is called the conditional probability and is denoted by P (A/B).

PERMUTATIONS AND COMBINATIONS

The word permutation means arrangement and combination means formation of groups.

Permutations

To understand the meaning of permutation i.e. arrangement, let us consider an example of four distinguishable objects marked a, b, c and d. Taking any two objects at a time, the possible arrangements are

ab, ba, ac, ca, ad, da, bc, cb, bd, db, cd, dc.

There are total 12 arrangements possible. In arranging these objects the order of their placing is also taken into consideration. Thus, 4 objects can be arranged in 12 ways by taking 2 objects at a time.

i.e., the number of permutation is 12.

$$^{4}P_{2} = 12$$

Ex- if 3 objects are taken at a time out of 4 objects a, b, c and d the various arrangements

abc	**abd**	**acd**	**bcd**
acd	**adb**	**adc**	**bdc**
bca	**bad**	**cda**	**cbd**
cab	**dab**	**dac**	**dbc**
cba	**dba**	**dca**	**dcb**

i.e.,

$$^{4}P_{3}=24$$

In general, the number of arrangements of n distinguishable objects by taking r at a time is given by

$$^{n}\mathbf{P_r} = \frac{\mathbf{n!}}{\mathbf{(n-r)!}} \qquad \text{.... (1)}$$

Thus,

$$^{4}P_{2} = \frac{\mathbf{4!}}{\mathbf{(4-2)!}}$$

$$= \frac{\mathbf{4!}}{\mathbf{2!}}$$

=4*3*2*1/2*1

=12

Combinations:

A combination is a selection of all or part of a set of objects, without regard to the order in which objects are selected.

For example, suppose we have a set of four letters: a, b, c and d. taking three at a time are

abc ,abd, acd, bcd

i.e.,only 4 combinations. Symbolically,

$$^{4}C_{3}=4$$

To calculate combinations, we will use the formula

$$^{n}C_{r} = \frac{n!}{r!\,(n-r)!} \qquad \ldots(2)$$

Where n represents the number of items, and r represents the number of items being chosen at a time.

But

$$^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$^{n}C_{r} = {}^{n}P_{r}/\,r!$$

$$^{n}P_{r} = r!\,.\,{}^{n}C_{r}$$

from eq.(1)

$$^{n}P_{n} = n! / (n - n)!$$

$$^{n}P_{n} = n! / 0!$$

$$= n!$$

$(\therefore 0! = 1)$

And form eq. (2)

$$^{n}C_{n} = n! / n! * (n - n)!$$

$$^{n}C_{n} = n! / n! * 0!$$

$$^{n}C_{n} = n! / n! \quad = 1$$

Reference books:

Statistical mechanics by **Satya Prakash**.

Relativity and statistical physics by **J.C. Agarwa**l.

Heat thermodynamics and statistical by **Dr. N. Subramaniam Brijlal**.