

M.G. KASHI VIDYAPITH, VARANASI
Three Years Degree Course Syllabus for

MATHEMATICS

(BASED ON UNIFORM SYLLABUS FOR U.P. STATE UNIVERSITIES)

For B.A./B.Sc. Classes(From 2012-13 onwards)

B.A./B.Sc. I

Paper I : ALGEBRA and TRIGONOMETRY M.M. : 65

Algebra

Unit 1. Sequence and its convergence (basic idea), Convergence of infinite series, Comparison test, ratio test, root test, Raabe's test, Logarithmic ratio test, Cauchy's condensation test, DeMorgan and Bertrand test and higher logarithmic ratio test. Alternating series, Leibnitz test, Absolute and conditional convergence, Congruence modulo m relation, Equivalence relations and partitions.

Unit 2. Definition of a group with examples and simple properties, Permutation groups, Subgroups, Centre and normalizer, Cyclic groups, Coset decomposition, Lagrange's theorem and its consequences.

Unit 3. Homomorphism and isomorphism, Cayley's theorem, Normal subgroups, Quotient group, Fundamental theorem of homomorphism, Conjugacy relation, Class equation, Direct product.

Introduction to rings, subrings, integral domains and fields, Characteristic of a ring, Homomorphism of rings, Ideals, Quotient rings.

Unit 4. Trigonometry

Complex functions and separation into real and imaginary parts, Exponential, direct and inverse trigonometric and hyperbolic functions, logarithmic function, Gregory's series, Summation of series.

Paper II : CALCULUS M.M. : 65

Differential Calculus

Unit 1. ϵ - δ definition of the limit of a function, Continuous functions and classification of discontinuities, Differentiability, Chain rule of differentiability, Rolle's theorem, First and second mean value theorems, Taylor's theorems with Lagrange's and Cauchy's forms of remainder, Successive differentiation and Leibnitz's theorem.

Unit 2. Expansion of functions (in Taylor's and Maclaurin's series), Indeterminate forms, Partial differentiation and Euler's theorem, Jacobians.

Unit 3. Maxima and Minima (for functions of two variables), Tangents and normals (polar form only), Curvature, Envelopes and evolutes.

Asymptotes, Tests for concavity and convexity, Points of inflexion, Multiple points, Tracing of curves in Cartesian and polar co-ordinates.

Unit 4. Integral Calculus

Reduction formulae, Beta and Gamma functions.

Quadrature, Rectification, Volumes and surfaces of solids of revolution, Pappus theorem, Double and triple integrals, Change of order of integration, Dirichlet's and Liouville's integral formulae.

Paper III : GEOMETRY and VECTOR CALCULUS

M.M. : 70

Geometry

Unit 1. General equation of second degree, Tracing of conics, System of conics, Confocal conics, Polar equation of a conic and its properties.

Unit 2. Three dimensional system of co-ordinates, Projection and direction cosines, Plane, Straight line.

Unit 3. Sphere, cone and cylinder.

Central conicoids, Reduction of general equation of second degree, Tangent plane and normal to a conicoid, Pole and polar, Conjugate diameters, Generating lines, Plane sections.

Unit 4. Vector Calculus

Vector differentiation and integration, Gradient, divergence and curl and their properties, Line integrals, Theorems of Gauss, Green and Stokes and problems based on these.

B.A./B.Sc. II**Paper I : LINEAR ALGEBRA and MATRICES**

M.M. : 65

Linear Algebra

Unit 1. Vector spaces and their elementary properties, Subspaces, Linear dependence and independence, Basis and dimension, Direct sum, Quotient space.

Unit 2. Linear transformations and their algebra, Range and null space, Rank and nullity, Matrix representation of linear transformations, Change of basis.

Unit 3. Linear functionals, Dual space, Bi-dual space, Natural isomorphism, Annihilators, Bilinear and quadratic forms, Inner product spaces, Cauchy-Schwarz's inequality, Bessel's inequality and orthogonality.

Matrices

Unit 4. Symmetric and skew-symmetric matrices, Hermitian and skew-Hermitian matrices, Orthogonal and unitary matrices, Triangular and diagonal matrices, Rank of a matrix, Elementary transformations, Echelon and normal forms, Inverse of a matrix by elementary transformations.

Characteristic equation, Eigen values and eigen vectors of a matrix, Cayley-Hamilton's theorem and its use in finding inverse of a matrix, Application of matrices to solve a system of linear (both homogeneous and non-homogeneous) equations, Consistency and general solution, Diagonalization of square matrices with distinct eigen values, Quadratic forms.

Paper II : DIFFERENTIAL EQUATIONS and INTEGRAL TRANSFORMS

M.M. : 65

Differential Equations

Unit 1. Formation of a differential equation (D.E.), Degree, order and solution of a D.E., Equations of first order and first degree : Separation of variables method, Solution of homogeneous equations, linear equations and exact equations, Linear differential equations with constant coefficients, Homogeneous linear differential equations,

Unit 2. Differential equations of the first order but not of the first degree, Clairaut's equations and singular solutions, Orthogonal trajectories, Simultaneous linear differential equations with constant coefficients, Linear differential equations of the second order (including the method of variation of parameters),

Unit 3. Series solutions of second order differential equations, Legendre and Bessel

functions (P_n and J_n only) and their properties.

Order, degree and formation of partial differential equations, Partial differential equations of the first order, Lagrange's equations, Charpit's general method, Linear partial differential equations with constant coefficients.

Partial differential equations of the second order, Monge's method.

Unit 4 . Integral Transforms

The concept of transform, Integral transforms and kernel, Linearity property of transforms, Laplace transform, Inverse Laplace transform, Convolution theorem, Applications of Laplace transform to solve ordinary differential equations.

Fourier transforms (finite and infinite), Fourier integral, Applications of Fourier transform to boundary value problems, Fourier series.

Paper III : MECHANICS

Dynamics

M.M. : 70

Unit 1. Velocity and acceleration along radial and transverse directions, and along tangential and normal directions, Simple harmonic motion, Motion under other laws of forces, Earth attraction, Elastic strings.

Unit 2. Motion in resisting medium, Constrained motion (circular and cycloidal only).

Unit 3. Motion on smooth and rough plane curves, Rocket motion, Central orbits and Kepler's law, Motion of a particle in three dimensions.

Unit 4. Statics

Common catenary, Centre of gravity, Stable and unstable equilibrium, Virtual work.

Forces in three dimensions, Poinsot's central axis, Wrenches, Null line and null plane.

B.A./B.Sc. III

Paper I : REAL ANALYSIS

M.M. : 75

Unit 1. Axiomatic study of real numbers, Completeness property in R , Archimedean property, Countable and uncountable sets, Neighbourhood, Interior points, Limit points, Open and closed sets, Derived sets, Dense sets, Perfect sets, Bolzano-Weierstrass theorem.

Unit 2. Sequences of real numbers, Subsequences, Bounded and monotonic sequences, Convergent sequences, Cauchy's theorems on limit, Cauchy sequence, Cauchy's general principle of convergence, Uniform convergence of sequences and series of functions, Weierstrass M -test, Abel's and Dirichlet's tests.

Sequential continuity, Boundedness and intermediate value properties of continuous functions, Uniform continuity, Meaning of sign of derivative, Darboux theorem.

Limit and continuity of functions of two variables, Taylor's theorem for functions of two variables, Maxima and minima of functions of three variables, Lagrange's method of undetermined multipliers.

Unit 3. Riemann integral, Integrability of continuous and monotonic functions, Fundamental theorem of integral calculus, Mean value theorems of integral calculus, Improper integrals and their convergence, Comparison test, μ -test, Abel's test, Dirichlet's test, Integral as a function of a parameter and its differentiability and integrability.

Unit 4. Definition and examples of metric spaces, Neighbourhoods, Interior points, Limit points, Open and closed sets, Subspaces, Convergent and Cauchy sequences, Completeness, Cantor's intersection theorem.

Paper II : COMPLEX ANALYSIS

M.M. : 75

Unit 1. Functions of a complex variable, Concepts of limit, continuity and differentiability of complex functions, Analytic functions, Cauchy-Riemann equations (Cartesian and polar form), Harmonic functions, Orthogonal system, Power series as an analytic function.

Unit 2. Elementary functions, Mapping by elementary functions, Linear and bilinear transformations, Fixed points, Cross ratio, Inverse points and critical points, Conformal transformations.

Unit 3. Complex Integration, Line integral, Cauchy's fundamental theorem, Cauchy's integral formula, Morera's theorem, Liouville theorem, Maximum Modulus theorem, Taylor and Laurent series.

Unit 4. Singularities and zeros of an analytic function, Rouché's theorem, Fundamental theorem of algebra, Analytic continuation.

Residue theorem and its applications to the evaluation of definite integrals, Argument principle.

Paper III : NUMERICAL ANALYSIS**Numerical Analysis**

M.M. : 75

Unit 1. Shift operator, Forward and backward difference operators and their relationships, Fundamental theorem of difference calculus, Interpolation, Newton-Gregory's forward and backward interpolation formulae.

Unit 2. Divided differences, Newton's divided difference formula, Lagrange's interpolation formula, Central differences, Formulae based on central differences : Gauss, Strling's, Bessel's and Everett's interpolation formulae, Numerical differentiation.

Unit 3. Numerical integration, General quadrature formula, Trapezoidal and Simpson's rules, Weddle's rule, Cote's formula, Numerical solution of first order differential equations : Euler's method, Picard's method, Runge-Kutta method and Milne's method, Numerical solution of linear, homogeneous and simultaneous difference equations, Generating function method.

Unit 4. Solution of transcendental and polynomial equations by iteration, bisection, Regula-Falsi and Newton-Raphson methods, Algebraic eigen value problems : Power method, Jacobi's method, Given's method, Householder's method and $Q-R$ method, Approximation : Different types of approximations, Least square polynomial approximation, Polynomial approximation using orthogonal polynomials, Legendre approximation, Approximation with trigonometric functions, exponential functions, rational functions, Chebyshev polynomials.

OPTIONAL PAPERS**Any one of the following papers :****M.M. : 75****Paper IV(a) : NUMBER THEORY and CRYPTOGRAPHY**

Unit 1. Divisibility : gcd, lcm, prime numbers, fundamental theorem of arithmetic, perfect numbers, floor and ceiling functions, Congruence : properties, complete and reduced residue systems, Fermat's theorem, Euler functions, Chinese remainder theorem.

Unit 2. Primality testing and factorization algorithms, Pseudo-primes, Fermat's pseudo-primes, Pollard's rho method for factorization.

Unit 3. Introduction to cryptography : Attacks, services and mechanisms, Security services, Conventional encryption - Classical techniques : Model, Steganography, Classical encryption technique, Modern techniques : DES, cryptanalysis, block cipher principles and design, Key distribution problem, Random number generation.

Unit 4. Hash functions, Public key cryptography, Diffie-Hellmann key exchange, Discrete logarithm-based crypto-systems, RSA crypto-system, Signature schemes, Digital signature standard (DSA), RSA signature schemes, Knapsack problem.

Elliptic curve cryptography : Introduction to elliptic curves, Group structure, Rational points on elliptic curves, Elliptic curve cryptography, Applications in cryptography and factorization, Known attacks.

Paper IV(b) : LINEAR PROGRAMMING

Unit 1. Linear programming problems, Statement and formation of general linear programming problems, Graphical method, Slack, and surplus variables, Standard and matrix forms of linear programming problem, Basic feasible solution.

Unit 2. Convex sets, Fundamental theorem of linear programming, Simplex method, Artificial variables, Big- M method, Two phase method.

Unit 3. Resolution of degeneracy, Revised simplex method, Sensitivity Analysis.

Unit 4. Duality in linear programming problems, Dual simplex method, Primal-dual method Integer programming.

Transportation problems, Assignment problems.

Paper IV(c) : DIFFERENTIAL GEOMETRY and TENSOR ANALYSIS**Differential Geometry**

Unit 1. Local theory of curves- Space curves, Examples, Plane curves, tangent and normal and binormal, Osculating plane, normal plane and rectifying plane, Helices, Serret-Frenet apparatus, contact between curve and surfaces, tangent surfaces, involutes and evolutes of curves, Intrinsic equations, fundamental existence theorem for space curves, Local theory of surfaces- Parametric patches on surface curve of a surface, surfaces of revolutions, Helicoids, metric-first fundamental form and arc length.

Unit 2. Local theory of surfaces (Contd.), Direction coefficients, families of curves, intrinsic properties, geodesics, canonical geodesic equations, normal properties of geodesics, geodesics curvature, geodesics polars, Gauss-Bonnet theorem, Gaussian curvature, normal curvature, Meusnier's theorem, mean curvature, Gaussian curvature, umbilic points, lines of curvature, Rodrigue's formula, Euler's theorem.

Unit 3. The fundamental equation of surface theory – The equation of Gauss, the equation of Weingarten, the Mainardi-Codazzi equation, Tensor algebra : Vector spaces, the dual spaces, tensor product of vector spaces, transformation formulae, contraction, special tensor, inner product, associated tensor.

Differential Manifold-examples, tangent vectors, connexions, covariant differentiation. Elements of general Riemannian geometry-Riemannian metric, the fundamental theorem of local Riemannian Geometry, Differential parameters, curvature tensor, Geodesics, geodesics curvature, geometrical interpretation of the curvature tensor and special Riemannian spaces.

Unit 4. Tensor Analysis

Contravariant and covariant vectors and tensors, Mixed tensors, Symmetric and skew-symmetric tensors, Algebra of tensors, Contraction and inner product, Quotient theorem,

Reciprocal tensors, Christoffel's symbols, Covariant differentiation, Gradient, divergence and curl in tensor notation.

Paper IV(d) : PRINCIPLES OF COMPUTER SCIENCE

Unit 1. Data Storage - Storage of bits, main memory, mass storage, Information of storage, The binary system, Storing integers, storing fractions, communication errors.

Data Manipulations - The central processing unit, The stored program concept, Programme execution, Other Architectures, arithmetic/logic instructions, Computer – peripheral communication.

Unit 2. Operating System and Network – The evolution of operating system, Operating system architecture, Coordinating the machine's activities, Handling competition among process, networks, network protocol.

Unit 3. Algorithms - The concept of an algorithm, Algorithm representation, Algorithm, Discovery, Iterative structure, Recursive structures, Efficiency and correctness, (algorithm to be implemented in C++).

Unit 4. Programming Languages - Historical perspective, Traditional programming Concepts, Program units, Languages implementation, Parallel computing, Declarative computing.

Software Engineering - The software engineering discipline, The software life cycle, Modularity, Development, Tools and techniques, Documentation, Software ownership and liability. **Data Structures** - Array, Lists, Stack, Queues, Trees, Customised data types, Object-oriented.

Paper IV(e) : DISCRETE MATHEMATICS

Unit 1. Propositional Logic - Proposition logic, basic logic, logical connectives, truth tables, tautologies, contradiction, normal forms (conjunctive and disjunctive), modus ponens and modus tollens, validity, predicate logic, universal and existential quantification.

Method of Proof - Mathematical induction, proof by implication, converse, inverse, contrapositive, negation, and contradiction, direct proof by using truth table, proof by counter example.

Unit 2. Relation - Definition, types of relation, composition of relations, domain and range of a relation, pictorial representation of relation, properties of relation, partial ordering relation.

Posets, Hasse Diagram and Lattices - Introduction, ordered set, Hasse diagram of partially ordered set, isomorphic ordered set, well ordered set, properties of lattices, and complemented lattices.

Boolean Algebra - Basic definitions, Sum of products and product of sums, Logic gates and Karnaugh maps.

Unit 3. Graphs - Simple graph, multi graph, graph terminology, representation of graphs, Bipartite, regular, planar and connected graphs, connected components in a graph, Euler graphs, Hamiltonian path and circuits, Graph colouring, chromatic number, isomorphism and homomorphism of graphs.

Tree - Definition, Rooted tree, properties of trees, binary search tree, tree traversal.

Unit 4. Combinatorics - Basics of counting, permutations, combinations, inclusion-exclusion, recurrence relations (n^{th} order recurrence relation with constant coefficients, Homogeneous recurrence relations, Inhomogeneous recurrence relations), generating function (closed form expression, properties of G.F., solution of recurrence relation using, G.F, solution of combinatorial problem using G.F.).

Finite Automata - Basic concepts of automation theory, Deterministic finite automation (DFA), transition function, transition table, Non deterministic finite automata (N DFA), Mealy and Moore machine, Minimization of finite automation.

Paper IV(A) : MATHEMATICAL STATISTICS

Probability Theory

Unit 1. Three definitions of probability (Mathematical, Empirical & axiomatic). Dependent, independent and compound events.

Addition and multiplication theorems of probability, conditional probability. Binomial and multinomial theorems of probability, Baye's theorem, Mathematical expectation and its properties, Moment generating functions (m.g.f.) and cumulants.

Distributions

Unit 2. Discrete distributions – Binomial & Poisson distributions and their properties.

Continuous distributions – Distribution function, Probability density function (Pdf), Cauchy's distribution, rectangular distribution, exponential distribution, Beta, Gamma Normal distributions and their properties.

Fitting of the Curves by method of least square – Straight line, parabola and exponential curves.

Correlation and Regression

Unit 3. Bivariate population, Meaning of correlation & regression. Coefficient of Correlation, rank correlation, lines of regression. Properties of regression coefficients, Partial and multiple correlation and their simple Properties.

Sampling Theory

Unit 4. Types of population, Parameters & Statistics, Null Hypothesis, Level of Significance, critical region. Procedure for testing Hypothesis. Type I & Type II error, χ^2 - distribution and its properties.

Simple and random sampling. Test of significance for large samples. Sampling distribution of Mean. Standard error, Test of significance based on χ^2 . Test of significance based on t, F & Z distribution, ANOVA.